DOON UNIVERSITY, DEHRADUN
School of Technology

Integrated MCA, First Semester, End Semester Examination, 2016
STM-502: Mathematics-I

Time Allowed: 3 Hours   *Maximum Marks: 50*

SECTION: A   **(Total Marks: 5 × 2 = 10)**

1. Fill up the blanks:
   a) Every equivalence relation on a set induces a unique _____ of the set.
   b) A linear system of equations, represented in the matrix form $\mathbf{Ax} = \mathbf{b}$, is inconsistent if _____.
   c) $f(x)$ is said to have a horizontal asymptote at $y = b$ if _____.
   d) The iterative formula to compute reciprocal of a number k using Newton's method is _____.

2. Differentiate the following functions w.r.t. x:

   a) $\sqrt{2x + (3x - 4x^2)^3}$ b) $(1 - 2x)^{\cosh x}$

3. Evaluate the following limits.

   a) $\lim\limits_{x \to 0} x^2 e^{\sin\left(\frac{100}{x}\right)}$ b) $\lim\limits_{x \to -\infty} \dfrac{\sqrt{4x^2 + 16}}{3 - 2x}$

4. Determine linear approximation to
   a) $\sin x$ at $x = 0$
   b) $3\sqrt[3]{x} - 1$ at $x = 8$

5. Approximate the area between $f(x) = -x^2 + 1$ and x-axis from $x = -2$ to $x = 2$ with 4 equal sub-divisions using:
   a) Left Riemann Sum
   b) Right Riemann Sum
   c) Mid-point Riemann Sum
   d) Trapezoidal Rule

SECTION: B   **(Total Marks: 5 × 4 = 20)**

1. a) Show that all positive integral powers of a symmetric matrix are symmetric.
   b) If $\mathbf{A}$ and $\mathbf{B}$ are symmetric matrices, show that $\mathbf{ABA}$ is also symmetric. Is $\mathbf{AB}$ always symmetric?

2. a) Given that $f(x)$ is continuous and differentiable in $[-10, -2]$, $f(-2) = -4$ and $f'(x) > -4$, find the range of values $f(-10)$ can take.
   b) Show that $f(x) = 7x^3 + 5x - 1$ has exactly one real root.

3. A light is on the top of a 12 ft tall pole and a 5 ft 6 in tall person is walking away from the pole at a rate of 2 ft/sec.
   a) At what rate is the tip of the shadow moving away from the pole when the person is 25 ft away from the pole?
   b) At what rate is the tip of the shadow moving away from the person when the person is 25 ft away from the pole?

4. Differentiate the following integrals at the given point.
   a) $\int_{x}^{45} \sqrt{|\cos t|}\, dt$ at $x = 4\pi/3$

   b) $\int_{x^2}^{2x^3} \left(-4t^2 + 2t\right) dt$ at $x = 1/2$

5. Define an improper integral. Evaluate the following improper integrals and classify each as convergent or divergent.
   a) $\int_{-\infty}^{\infty} \frac{250}{25 + t^2} dt$

   b) $\int_{0}^{1/2} \frac{1}{(2t-1)^2} dt$

SECTION: C   **(Total Marks: 4 × 5 = 20)**

1. a) Solve the following non-homogeneous linear system using Gauss Elimination and Back Substitution:
   $3.0x_1 + 2.0x_2 + 2.0x_3 - 5.0x_4 = 8.0 \quad 0.6x_1 + 1.5x_2 + 1.5x_3 - 5.4x_4 = 2.7 \quad 1.2x_1 - 0.3x_2 - 0.3x_3 + 2.4x_4 = 2.1$
   b) Find the eigenvalues and eigenvectors of the following matrix:
   $$A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$$

2. Find and classify all the critical points of the following function. Also determine the intervals in which the function is increasing and decreasing.
   $$f(x) = x \sqrt[3]{x^2 - 4}$$

3. Explain Newton's method to approximate solution to $f(x) = 0$. Use the method to find approximations to the following correct to 6 decimal places.
   a) $\sqrt[3]{3}$
   b) Point of intersection of $y = 2x$ and $y = \cos x$

4. Evaluate the following definite integrals using Riemann sum.
   a) $\int_{-5}^{-1} \left(x^3 + 2x^2 - 5\right) dx$

   b) $\int_{0}^{10} 10e^{2x}\, dx$