3/6/2024

DOON UNIVERSITY, DEHRADUN

End Semester Examination IV Semester, 2024

Academic Year 2023-24 (Even Semester)

School of Physical Science, Department Name: Mathematics

Programme Name: Integrated M.Sc.

Course Code with Title: MAC:253, Ring Theory

Duration: 2.0 hours
Maximum Marks: 50

SECTION A: Attempt all. **(Total Marks:1X10=10)**

1. Show that $1-i$ is irreducible in $Z[i]$.
2. Determine $f(x)=x^2+x+1$ is irreducible over Z_4.
3. Define Unique Factorization Domain.
4. Define Euclidean Domain.
5. Is $Z_2[x]/<x^3+x^2+1>$ a field. Find the cardinality of it.
6. the polynomial $f(x)=x^2+5$ is
   a) irreducible over C
   b) irreducible over R
   c) irreducible over Q
   d) not irreducible over Q
7. The number of roots of the polynomial x^3-x in Z_6?
8. Show that $\langle x^2+1\rangle$ is the maximal ideal in R[x].
9. Show that $3x^4+9x^3-7x^2+15x+25$ is irreducible over Q.
10. Define Principal ideal domain. Give example.

SECTION B: Attempt any 5. **(Total Marks: 5X4=20)**

11. Let F be a field. Then show that F[x] is a Principal Ideal Domain.
12. State and prove remainder theorem.
13. Show that in a PID an element is prime iff it is irreducible.
14. Show that 3 is a irreducible element of $Z\sqrt{-5}$.
15. Prove that an element 'a' in a field F is a zero of f$f(x)\in F[x]\ if\ f(x-a)$ is a factor of $f(x)$ in $F[x]$.
16. Prove that $Z\left[\sqrt{-3}\right]$ is not a Principal ideal domain.

SECTION C: Attempt any 4. **(Total Marks:4X5=20)**

17. State and prove Division Algorithm.
18. Check if $f(x)=x^5+2x+4$ is irreducible over Q.
19. Define associates. Let D be an integral domain. Define $a\sim b\ if\ a\ and\ b$ are associates. Show that this defines an equivalence relation on D.

20. Define ring homomorphism. Show that $\emptyset: x \rightarrow 5x$ from Z_4 to Z_{10} is a ring Homomorphism.
21. Define irreducible polynomials. Let F be a field and p(x) ∈ F[x]. then show that <p(x)> is a maximal ideal in F[x] if and only if p(x) is irreducible over F.