

SECTION: C

III. Attempt any two:

2X10=20

- a. (i). Prove that the external direct product of a finite number of groups is a group under component-wise product. Find the order and inverse (2,3) in $Z_3 \oplus Z_5$. Is $Z_3 \oplus Z_6$ cyclic? justify your answer. (7)
- (ii). $f: (G, +) \rightarrow (G', *)$, $f(x) = 2^x$. Show that f is a homomorphism. (3)
- b. (i). Prove that a homomorphism ϕ of a group is injective iff $\ker \phi = \langle e \rangle$. (6)
- (ii). Determine all homomorphism from Z_{20} to Z_8 . (4)
- c. (i). Define factor group. Let $N \triangleleft G$ and $H < G$, containing N, then show that $H/N < G/N$. (5)
- (ii) Draw the Cayley table for $S_3 / \langle (123) \rangle$. (5)

