14/12/23

DOON UNIVERSITY, DEHRADUN

End Semester Examination, Second Semester, 2023-24

Academic Year 2023-2024(Odd Semester)

School of Physical Sciences

Class: B.Sc. (Hons)Mathematics | **Course: PDE & System of ODEs**

Semester: III | **Course Code: MAC-202, Credit: 04**

Time Allowed: 2Hours | ***Maximum Marks: 50***

Note: **Attempt all five questions in Section A. Each question carries 2 marks.**
Attempt any four out of five questions in Section B. Each question carries 5 marks.
Attempt any two out of three questions in Section C. Each question carries 10 marks.

SECTION: A
(Very Short Answer Type Questions)

(Marks:5X2=10)

1. Given non- linear pde of first order *F(x,y,z,p,q)=0.*, write characteristic equations corresponding to it.
2. Form the partial differential equation by eliminating arbitrary function from $z = xy + f(x^2 + y^2)$.
3. Write short note on Clairaut's form of first order pde.
4. Given second order linear PDE in two variables x *and* y:
   $A\,u_{xx} + B\,u_{xy} + Cu_{yy} + Du_x + Eu_y + Fu = G$ ……(1)
   where the coefficients $A, B, C, D, E, F,$ *and* G in (1) may be functions of x *and* y in the domain Ω. Describe all the possible natures with conditions that (1) can show.
5. Write short note on boundary value problems.

SECTION: B
(Short Answer Type Questions)

(Marks: 4X5=20)

6. Derive the partial differential equation by eliminating arbitrary function ϕ from the equation $\phi(u, v) = 0$ where u and v are functions of x , y and z .
7. Given $3u_{xx} + 10u_{xy} + 3u_{yy} = 0$. Determine its nature, and characteristic differential equation, and family of characteristic lines, canonical transformations and canonical form of the equation. and solve it.
8. Find the solution of the equation $(D^2 + 2D\,D' + D'^2 - 2D - 2D')u = \sin(x + 2y)$.
9. Show that $(x, t) = \frac{1}{\sqrt{4\pi\alpha t}} \exp[-(x-\xi)^2 \frac{1}{4\alpha t}]$, where ξ is an arbitrary real constant , is a solution of $\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$, $-\infty < x < \infty$, $t > 0$.
10. Solve the following pde by the method of separation of variables:
    $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$.

SECTION: C

(Long Answer Type Questions)

11. Derive the one-dimensional wave equation.

12. Describe the Picard's method of successive approximations to solve a first order ordinary differential equation with initial condition.

Solve the following using Picard's method of successive approximations:

$\frac{dy}{dx} = x^2 + y^2, y(0) = 0$. Find $y(0.4)$.

13. Describe the modified Euler's method to solve a first order ordinary differential equation with initial condition.

Using the modified Euler's method solve the following:

$\frac{dy}{dx} = \frac{-x}{y}, y(0) = -1$, using h=0.1, and find $y(0.4)$.