15/12/23

DOON UNIVERSITY, DEHRADUN
Department of Mathematics, School of Physical Sciences
Final Semester Examination, ODD Semester 2023-2024

Class : B.Sc (Honours/ with Research) Mathematics
Course: Logic Sets and Special functions
Time Allowed : 2 Hours

Semester : 1
Course Code: MAC-103
Max Marks : 50

Note: Attempt all questions in Section A. Each question carries **2** marks.
Attempt all questions in Section B. Each question carries **5** marks.
Attempt any **Two** questions in Section C. Each question carries **10** marks.

Section: A
(Very Short Answer Type Questions)

Attempt all Five questions. [5×2 = 10Marks]

1. If p is true and q is false, find the truth values of the following:
   (a) $\sim (p \rightarrow \sim q)$ (b) $(p \wedge q) \rightarrow (p \vee q)$ (c) $\sim (p \wedge q) \vee \sim (q \rightarrow p)$ (d) $(p \rightarrow q) \vee \sim (p \leftrightarrow \sim q)$

2. Fill in the blanks
   (a) A _____ is an ordered collection of objects.
   (b) The set of positive integers is _____.
   (c) If A has m elements and B has n elements, then $A \times B$ has elements _____.
   (d) Every set is a _____ of itself.

Multiple Choice Questions:

3. If A is not equal to B, then the Cartesian product
   A) $A \times B$ not equal $B \times A$
   B) is not possible
   C) $A \times B > B$ or E$< A$
   D) None of the above

4. The relation $R = \{(1,1),(2,2),(3,3),(1,2),(2,3),(1,3)\}$ on set $A = \{1,2,3\}$ is
   A) Reflexive but not Symmetric
   B) Reflexive but not Transitive
   C) Symmetric and Transitive
   D) Neither symmetric nor Transitive

5. If $R = (1,1),(2,3),(4,5)$, then domain of the function is
   A) Dom $R = \{2,3,4,5\}$
   B) Dom $R = \{1,1,4,5\}$
   C) Dom $R = \{1,3,5\}$
   D) Dom $R = \{1,2,4\}$

Section: B

(Short Answer Type Questions)

Attempt all four questions. [4×5 = 20 Marks]

6. Define converse, inverse and contrapositive of conditional statement. Also show their truth table . Write converse, inverse and contrapositive of the following:

   (a) Indian team wins whenever match is played in Kolkata Home town of Ganguly.

   (b) If $3y - 2 = 10$, then $x = 1$.

   (c) If two angles are congruent, then they have the same measure.

7. Associative composition of relation: Let A, B, C and D be sets. Suppose R is a relation from A to B and S be a relation from B to C and T be a relation from C to D. Then show that,

$$(RoS)oT = Ro(SoT)$$

8. (i) Given that $A = \{1, 2, 3, 4\}$ and $B = \{x, y, z\}$. Let R be the following relation from $A$to B:

$$R = \{(1, y), (1, z), (3, y), (4, x), (4, z)\}$$

   (a) Determine the matrix of the relation.

   (b) Draw the arrow diagram of R.

   (c) Find the inverse relation R^{-1} of R. (d) Determine the domain and range of R.

   (ii) Let $A = \{1, 2, 3, 4, 5, 6\}$ and let R be the relation on A defined by "x divides y".

   (a) Write R as a set of ordered pairs.

   (b) Draw it digraph.

   (c) Find the inverse relation R^{-1} of R.

9. In a survey, it is found that 21 people read English newspaper, 26 people read Hindi newspaper, and 29 people read regional language newspaper. If 14 people read both English and Hindi newspapers; 15 people read both Hindi and regional language newspapers; 12 people read both English and regional language newspaper and 8 read all types of newspapers, find:

   (i) How many people were surveyed?

   (ii) How many people read only regional language newspapers?

Section: C

(Long Answer Type Questions)

Attempt any Two questions. **[2× 10 = 20 Marks]**

10. Define the equivalence relation in detail. If R is relation on the set of integers such that $(a, b) \in \mathbb{R}$ iff $3a + 4b = 7n$ for some integer. Prove that R is an equivalence relation.

11. If $f : A \to B$ and $g : B \to C$ be one-one and onto function then prove that gof is one-one and onto and Show that

$$(gof)^{-1} = f^{-1}og^{-1}$$

12. Using the principle of mathematical induction, prove that

$$1 \times 3 + 3 \times 5 + 5 \times 7 + \ldots + (2n - 1)(2n + 1) = \tfrac{1}{3}n(4n^2 + 6n - 1).$$