

21/12/23

**DOON UNIVERSITY, DEHRADUN**  
**Mid Semester Examination, Int. B.Sc. Third (3<sup>rd</sup>) Semester, 2023**  
**Academic Year 2023-24 (Odd Semester)**  
**School of Physical Sciences: Department Mathematic**  
**Programme: Generic Mathematics (Application of Algebra)**  
**Course Code with Title: MAG-201 Application of Algebra**

**Time Allowed 2.00 Hours**

**Maximum Marks: 30**

The section A contains six questions each of one mark; all are compulsory. The section B contains five questions each of three marks; any four are to be attempted. The section C contains three questions, each of six marks; any two are to be attempted.

**SECTION: A**  
**(Very Short Answer Type Questions)**

1. Explain the parameters of BIBD.
2. Prove: A BIBD with parameters  $(v, k, \lambda)$  is symmetric if and only if  $v = k + \lambda$ .
3. Show that it is not possible to have a BIBD with parameters  $(v, k, \lambda)$ , if  $v = 12, k = 5$ .
4. Given a subset  $S$  of a group  $G$  and an element  $g$  in  $G$ , Then define coset  $g + S$ .
5. Define  $t$ -fold  $(v, k, \lambda)$ -difference set family in an additive group  $G$ .
6. Define a commutative group, and a field.

**SECTION : B**  
**(Short Answer Type Questions)**

7. Let  $V$  be a set of  $v$  elements, and let  $\mathcal{D}$  be the set of all subsets of  $V$  having  $k$  elements,  $1 < k < v$ . Then  $\mathcal{D}$  is a balanced incomplete block design on  $V$  with parameters  $(v, k, \lambda)$  where  $\lambda = \frac{k(k-1)}{v-1}$ .
8. Let  $\mathcal{D}$  be a symmetric BIBD with parameters  $(v, k, \lambda)$ , and  $D = \{B_1, B_2, B_3, B_4, B_5, B_6, B_7\}$ , where  $B_1 = \{1, 2, 4\}, B_2 = \{2, 3, 5\}, B_3 = \{3, 4, 6\}, B_4 = \{4, 5, 7\}, B_5 = \{5, 6, 1\}, B_6 = \{6, 7, 2\}, B_7 = \{7, 1, 3\}$ . Find the dual of the symmetric BIBD  $\mathcal{D}$ .
9. Let  $G$  be the additive group of integers modulo 7. Let  $S = \{1, 2, 4\}$ . Show that  $S$  is a difference set in  $G$ , and find its parameters.
10. Let  $S$  be a difference set in a group  $G$  with parameters  $(v, k, \lambda)$ . Then  $v - k + \lambda = k$ .
11. Find the set of all quadratic residues modulo 7.

**SECTION: C**  
**(Long Answer Type Questions)**

12. Find the set of quadratic residues modulo 11, and construct the symmetric BIBD determined by it.
13. Let  $G = \mathbb{Z}_9, S_1 = \{0, 1, 2, 4\}$ , and  $S_2 = \{0, 3, 4, 7\}$ . Show that  $S_1, S_2$  form a difference set family, and find its parameters. Construct the BIBD  $D$  induced by the difference set family of  $S_1, S_2$ . What are the parameters of  $D$ ?
14. Find a 2-fold difference set family in the additive group  $\mathbb{Z}_{13}$ . What are the parameters of the BIBD induced by it?