DOON UNIVERSITY, DEHRADUN

End Semester Examination, III Semester, 2023

Academic Year 2023-24 (Odd Semester)

School of Physical Sciences Department of Mathematics

Programme: BSc Mathematics

Course Code with Title: MAC-203: Mathematical Transforms.

Time Allowed 2.00 Hours *Maximum Marks: 50*

All questions are compulsory.

SECTION: A [5x3=15M]

Q1. Find the Laplace transform of $e^{-4t}\int_0^t t\sin 3t\,dt$.

Q2. Find the inverse Laplace transform of $\frac{1}{(s+1)^2}$.

Q3. Find the z-transform of the ka^k, $k \geq 0$.

Q4 Find the z-transform of the $f(x) = \frac{a^k}{k!}$, $k \geq 0$.

Q5 Find the Fourier transform of $f(x) = \begin{cases} x & 0 < x < a \\ 0 & otherwise \end{cases}$

SECTION: B [3x5=15M]

Q6. Find the inverse Laplace transform of $\frac{s^2+1}{(s+1)(s-2)^2}$.

Q7. If $f(x) = \begin{cases} 1 & |x| < a \\ 0 & |x| > a \end{cases}$

Using Fourier transform find the value of $\int_0^\infty \frac{\sin x}{x}\,dx$.

Q8. Find the inverse z transform of $\frac{z}{(z-2)(z-3)}$, $|z| > 3$

OR

Find the inverse z-transform of $\frac{1}{(z-a)^2}$

i. $|z| > |a|$

ii. $|z| < |a|$

SECTION: C [2x10=20M]

Q9. Find the Fourier cosine and sine transforms of

$$f(x) = \begin{cases} x & 0 < x < 1 \\ 2-x & 1 < x < 2 \\ 0 & x > 2 \end{cases}$$

Q10. Solve using Laplace transform

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 8y = 1, \qquad y(0) = 0, y'(0) = 1$$

OR

$$\frac{dy}{dx} + y = \cos 2t\ , y(0) = 1$$