Roll No......................

13/12/23

BSc Honours with Research (Mathematics)First Semester
End Semester Examination, 2023-2024
Department of Mathematics, SOPS, Doon University, Dehradun
Core Course: MAC102, Algebra

Time Allowed: 02 Hours *Maximum Marks: 50*

Note: Do neat and clean work.

SECTION A

Attempt ALL Questions. **(2x5=10)**

1. Find the rank and the basis of the row space of the matrix $\begin{bmatrix} 1 & 2 & 0 & -1 \\ 2 & 6 & -3 & -3 \\ 3 & 10 & -6 & -5 \end{bmatrix}$.
2. Show that the vectors u=(1,1,1), v=(1,2,3) and w=(1,5,8) span R^3.
3. Prove that if A is a skew-symmetric matrix of odd order then $|A| = 0$and hence its inverse does not exist.
4. Find the value of α so that the vectors (1,2,9,8), (2,0,0, α),(α,0,0,8) and (0,0,1,0) are linearly dependent.
5. Define singular and non-singular linear transformation.

SECTION B

Attempt any FOUR **(5x4=20)**

7. Find the value of θ for which the system of equations $2\sin\theta . x + y - 2z = 0$, $3x + 2\cos2\theta . y + 3z = 0$, $5x + 3y - z = 0$ has a non-trivial solution.
8. Show that the set of vectors (1, -1, 0), (2, 3, -2) and (-2, 0, 1) are linearly independent.
9. Let V be a vector space of n-square real matrices. Let M be an arbitrary but fixed matrix in V. Let $T: V \to V$ be define as $T(A) = AM + MA$, where A is any matrix in V. Show that T is linear.
10. State and prove Cayley-Hamilton's theorem . Verify it for the matrix A= $\begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$.
11. Define a composite number and prove that if $2^n - 1$ is prime, then n is prime.

SECTION C

Attempt any FOUR **(5x4=20)**

11. For what values of k, the equations , $(x + y + z = 1;\ \ 2x + y + 4z = k;\ 4x + y + 10zt = k^2)$ have a solution and solve them completely in each case.
12. State and prove rank-nullity theorem for a linear transformation $T: V \to U$.
13. Let T be a linear transformation on R^3, define by $T(x, y, z) = (3x, x - y, 2x + y + z)$, show that T is invertible and find the rule by which inverse of T is defined.
14. A necessary and sufficient condition for a non empty subset W of a vector space V(F) to be a subspace is that W is closed under addition and scalar multiplication, prove this statement.
15. Let T be a linear transformation on R^3, the matrix of which in the standard ordered basis is, A= $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$, find a basis for the range of T and basis for the null space of T. Also find the dimension of image and dimension of kernel of T.
