Roll No:__________

DOON UNIVERSITY, DEHRADUN

Department of Mathematics, School of Physical Sciences

END Semester Examination, ODD Semester, Session 2023-2024

Class : B.Sc.(Hons/with Research) Mathematics
Course: Real Analysis-2
Time Allowed : 2 Hours

Semester : III
Course Code: MAC201
Max Marks : 50

Note: Read the instructions carefully given at the beginning of each section.

Section: A

(Very Short Answer Type Questions)

Attempt all Four questions. Each question carries 3 marks. [4×3 = 12 Marks]

1. Let $D \subset R$ be a compact set and a function $f : D \to R$ be continuous on D, then prove that $f(D)$ is a compact set in R.

2. Examine the continuity and differentiability of the function $f(x) = |\sin \frac{2\pi x}{L}|, (-\infty < x < \infty, L > 0)$ at $x = 0$.

3. Which of the following function is uniformly continuous on $(0, \pi)$. Explain with proper logic.
(a) $x \cos \frac{1}{x}$ (b) e^x (c) $\frac{e^x}{x}$ (d) $\frac{1}{e^x}$.

4. Evaluate $\lim\limits_{x \to \infty} \sqrt{x^2 + x - 1} - x$.

Section: B

(Short Answer Type Questions)

Attempt any Four questions. Each question carries 5 marks. [4×5 = 20 Marks]

5. State and prove sequential criterion for finite limit of a function.

6. State and prove Rolle's theorem with its geometrical interpretation. Check its applicability for the function $f(x) = 1 - (x-1)^{2/3}$ on the interval $[0, 2]$.

7. Obtain Maclaurin's series expansion of $(1+x)^m, \quad -1 < x < 1, \quad m \in R$.

8. Let I be an open interval and let $f : I \to R$ have a second derivative on I, then prove that f is convex if and only if $f''(x) \geq 0; \quad \forall x \in I$.

9. State and prove L'Hospital rule for 0/0 indeterminate form. Also, find the value of $\lim\limits_{x \to 0} (\cos x)^{1/x^2}$.

Section: C

(Long Answer Type Questions)

Attempt any Three questions. Each question carries 6 marks. [3×6 = 18 Marks]

10. A function f is defined as $f(x) = \begin{cases} x^p \cos \frac{1}{x}, & \text{when } x \neq 0 \\ 0, & \text{when } x = 0. \end{cases}$
What condition should be imposed on p so that:
(i) f may be continuous at $x = 0$?
(ii) f may be derivable at $x = 0$?

11. State and prove Lagranges's mean value theorem. Hence, prove that $x - \frac{x^2}{2} + \frac{x^3}{3(1+x)} < \log(1+x) < x - \frac{x^2}{2} + \frac{x^3}{3}$ for $x > 0$.

12. State and prove Taylor's theorem with Schlömilch and Röuche form of remainder.

13. State and prove the necessary and sufficient(second derivative test) conditions for the existence of extreme values (max / min) of a function.