26-3-18

DOON UNIVERSITY, DEHRADUN
Mid Semester Examination, 2017-2018
School of Physical Sciences
B.Sc.(Mathematics)VI Semester
Course: MAD-356, Differential Geometry

Time Allowed: 2 Hours Maximum Marks: 30

***Note:***
*Attempt all questions from Section A, any four questions from Section B and any two questions from Section C.*

**Section: A** $(1.5 \times 4 = 6$ Marks)

(1) Define the osculating plane, normal plane and the rectifying plane.
(2) Compute the quantity $[\mathbf{r}', \mathbf{r}'', \mathbf{r}''']$ in terms of $\kappa$ and $\tau$.
(3) Find the arc length of the circular helix

$$\mathbf{r}(u) = (a\cos u, a\sin u, cu), u \in (-\infty, \infty)$$

from $(a, 0, 0)$ to $(a, 0, 2c)$.
(4) Calculate the torsion $\tau$ of the curve $\mathbf{r} = (u, u^2, u^3)$.

**Section: B** $(3 \times 4 = 12$ Marks)

(1) Establish the formula

$$s = \int_a^t |\dot{\mathbf{r}}|\, dt$$

for a parametric curve $\mathbf{r} = \mathbf{r}(t)$, where $t = a$ is the initial point.
(2) For any curve prove that

$$[\mathbf{t}', \mathbf{t}'', \mathbf{t}'''] = \kappa^3(\kappa\tau' - \tau\kappa').$$

(3) Show that the osculating plane at any point to the curve has three point contact with the curve.
(4) Show that the necessary and sufficient condition that a curve be a straight line is that $\kappa = 0$ at all point.

**Section: C** $(6 \times 2 = 12$ Marks)

(1) Establish the equation of osculating plane at a point $s$ of the curve $r = \mathbf{r}(s)$. Hence, find the same for the curve $\mathbf{r}(u) = (3u, 3u^2, 2u^3)$ at point $u$.
(2) Find the arc length of the curve

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1, \quad x = a\cosh\left(\frac{z}{a}\right)$$

from $(a, 0, 0)$ to $(x, y, z)$.
(3) Prove the Serret Frenet formulas: $\mathbf{t}' = \kappa\mathbf{n}$, $\mathbf{n}' = \tau\mathbf{b} - \kappa\mathbf{n}$, $\mathbf{b}' = -\tau\mathbf{n}$.