Roll No:______________

22-3-2018

DOON UNIVERSITY, DEHRADUN

Department of Mathematics, School of Physical Sciences

Mid Semester Examination, Even Semester 2017-18

Class : M.Sc. Mathematics | **Semester : IV**

Course: Fuzzy Sets and Logics | **Course Code: MAC-551**

Time Allowed : 2 Hours | **Max Marks : 30**

Note: Attempt all **Three** questions in Section A. Each question carries **2** marks.
Attempt any **Three** questions in Section B. Each question carries **4** marks.
Attempt any **Two** questions in Section C. Each question carries **6** marks.

Section: A

(Short Answer Type Questions)

Attempt all Three questions. **[3×2 = 6 Marks]**

1. Define the following terms:
   (a) Fuzzy complement (b) t-norm (c) Convex fuzzy set (d) Support of a fuzzy set.
2. If $X = \{a, b, c, d\}$, $\tilde{A} = \{(a, 0.8), (b, 1.0), (c, 0.3), (d, 0.1)\}$ and $\tilde{B} = \{(a, 0.2), (b, 0.5), (c, 0.7), (d, 0.9)\}$ then find α-cut sets A_α and B_α for $\alpha = 0.3$.
3. Calculate the degree of subsethood $S(\tilde{A}, \tilde{B})$ and $S(\tilde{B}, \tilde{A})$ for the fuzzy sets $\tilde{A} = \{(x, 1.0), (y, 1.0), (z, 1.0)\}$ and $\tilde{B} = \{(v, 0.4), (w, 0.2), (x, 0.5), (y, 0.4), (z, 1.0)\}$.

Section: B

(Short Answer Type Questions)

Attempt any Three questions. **[3×4 = 12 Marks]**

4. Show that $\tilde{A} \cup (\tilde{A} \cap \tilde{B}) = \tilde{A}$.
5. Let $\tilde{A} = (1, 3, 5)$ and $\tilde{B} = (5, 7, 10)$ be two triangular fuzzy numbers then find (a) $A \oplus B$ (b) $A \ominus B$ (c) $A \odot B$ (d) $A \oslash B$. Also express $\tilde{A}$ and $\tilde{B}$ interms of LR-type fuzzy numbers.
6. Consider two fuzzy sets $\tilde{A} = \{(0, 0), (20, 0.5), (40, 0.65), (60, 0.85), (80, 1.0), (100, 1.0)\}$ and $\tilde{B} = \{(0, 0), (20, 0.45), (40, 0.6), (60, 0.8), (80, 0.95), (100, 1.0)\}$. Find $\tilde{A} \cup \tilde{B}, \tilde{A} \cap \tilde{B}, \tilde{A}^c \cup \tilde{B}, \tilde{A}^c \cap \tilde{B}$.
7. Let $\tilde{A} = \{(-1, 0.5), (0, 1.0), (1, 0.5), (2, 0.3)\}$ and $\tilde{B} = \{(2, 0.5), (3, 1.0), (4, 0.5), (5, 0.3)\}$ be two fuzzy sets defined on the universal set $X = Z$. Let a function $f : X \times X \to X$ be defined for all $x_1, x_2 \in X$ by $f(x_1, x_2) = x_1 x_2$. Calculate $f(\tilde{A}, \tilde{B})$ using extension principle.

Section: C

(Long Answer Type Questions)

Attempt any Two questions. **[2×6 = 12 Marks]**

8. Prove that a fuzzy set $\tilde{A}$ in X is convex iff for any $\alpha \in [0, 1]$, A_α is convex. Also show that the fuzzy set $\tilde{A}$ with following membership function is convex.
   $$\mu_{\tilde{A}}(x) = \begin{cases} 0 & , x \leq 10 \\ \frac{1}{1+(x-10)^{-2}} & , x > 10 \end{cases}$$
9. Let i be a t-norm and c be an involutive fuzzy complement. Then the binary operation u defined by $u(x, y) = c\big(i(c(x), c(y))\big), \forall x, y \in [0, 1]$ is a t-conorm and $< i, u, c >$ is a dual triple.
10. (a) Show that $g_\lambda(x) = \begin{cases} \frac{1}{\lambda} \log(1 + \lambda x) & , \lambda > -1, \lambda \neq 0 \\ x & , \lambda = 0 \end{cases}$ for $x \in [0, 1]$ is increasing generator for a fuzzy complement c_λ. Hence find c_λ.

    (b) Consider the fuzzy set $\tilde{A} = \{(x_1, 0.2), (x_2, 0.4), (x_3, 0.6), (x_4, 0.8), (x_5, 1.0)\}$. Show that this fuzzy set can be represented by its α-cuts.