Roll No:_____

28-3-2018

DOON UNIVERSITY, DEHRADUN
Department of Mathematics, School of Physical Sciences
Mid Semester Examination, Even Semester 2017-18

Class : M.Sc.Mathematics | **Semester : II**
Course: Measure and Integration | **Course Code: MAC-454**
Time Allowed : 2 Hours | **Max Marks : 30**

Note: Attempt all **Six** questions in Section A. Each question carries **1** marks.
Attempt any **Four** questions in Section B. Each question carries **3** marks.
Attempt any **Three** questions in Section C. Each question carries **4** marks.

Section: A
(Very Short Answer Type Questions)

Attempt all Six questions. **[6×1 = 6 Marks]**

1. The cardinal number of the set whose elements are the roots of a polynomial equation with integer coefficients is:
(a)finite (b) χ_0 (c) f (d) c.

2. If E is any given set then given $\epsilon > 0$, there exists an open set $O \supseteq E$ such that
(a)$m^*(O) < m^*(E) + \epsilon$ (b) $m^*(E) < m^*(O) + \epsilon$ (c)$m^*(O) < m^*(E) - \epsilon$ (d) none of these.

3. Which of the following relation shows that each of the interval $[a, b], (a, b], (a, b), [a, b)(a < b)$ has the power of continuum
(a) $f(x) = a + (b - a)x$ (b) $f(x) = b + (a - b)x$ (c) $f(x) = a + bx$ (d) $f(x) = b + ax$.

4. If E_1 and E_2 are any measurable sets such that $E_1 \subset E_2$ and $m(E_2) < \infty$ then
Is $E_2 - E_1$ measurable? If yes then $m(E_2 - E_1)$ is.....

5. The cardinal number of the sets $\{\phi, \{\phi\}, \{\phi, \{\phi\}\}\}$ is
(a)4 (b) 3 (c) χ_0 (d) c.

6. According to Schroeder-Bernstein theorem: $A \sim B \quad iff$.....

Section B
(Short Answer Type Questions)

Attempt any Four questions. **[4×3 = 12 Marks]**

7. Prove or disprove that the set of irrational numbers in $[0, 1]$ is non-denumerable. Also, find its cardinal number.

8. Define the concept of exterior measure of a set.

9. Prove that E is measurable if $m^*(E) = 0$.

10. Show that the union of denumerable collection of denumerable sets is denumerable.

11. show that $2^{\chi_0} = c$

Section C

(Long Answer Type Questions)

Attempt any Three questions. **[3×4 = 12 Marks]**

12. Define Cantor set and show that it is measurable and find its measure.

13. State and prove of the Cantor's theorem.

14. Show that every infinite set X contains a subset which is denumerable.

15. State and prove Schroeder-Bernstein theorem.