23/3/2018

DOON UNIVERSITY, DEHRADUN

Mid Semester Examination, Second Semester, 2017-18

School of Physical Sciences

Class: M.Sc. Mathematics   Course: Partial Differential Equations

Semester: II Course Code: MAC-453

Time Allowed: 2Hours   Maximum Marks: 30

Note: Attempt all six questions in Section A. Each question carries 1 marks.

Attempt any four questions in Section B. Each question carries 3 marks.

Attempt any two questions in Section C. Each question carries 6 marks.

SECTION: A

(Very Short Answer Type Questions)

(Marks:6X1=6)

1. Form the partial differential equation by eliminating h and k from the equation
   $(x-h)^2 + (y-k)^2 + z^2 = \lambda^2$.

2. Write short note on classification of first order partial differential equations into linear ,semi-linear, quasi-linear, and non-linear differential equations with examples.

3. Solve the following equation $p^2 + q^2 = x + y$.

4. What is Pfaffian differential equation?state necessary and sufficient condition for a differential equation $Pdx + Qdy + Rdz = 0$ to be exact.

5. Write short note on orthogonal trajectories of a system of curves on a surface.

6. How can a linear partial differential equation of second order :
   $Au_{xx} + Bu_{xy} + Cu_{yy} + Du_x + Eu_y + Fu = G = 0$ be classified?

SECTION: B

(Short Answer Type Questions)

(Marks: 4X3=12)

7. Solve the following system of equations: $\dfrac{adx}{(b-c)yz} = \dfrac{bdy}{(c-a)zx} = \dfrac{cdz}{(a-b)xy}$.

8. Solve $(2x^2 + 2xy + 2xz^2 + 1)dx + dy + 2zdz = 0$.

9. Using Charpits method solve the following equation $z^2 = pqxy$.

10. Solve the following equations by Jacobi'smethod . $2p_1 x_1 x_3 + 3p_2 x_3^2 + p_2^2 p_3 = 0$

11. Show that following equations are compatible and solve them: $xp = yq, z(xp + yq) = 2xy$.

12. Derive Lagrange's identity for the linear second order pde.

SECTION: C

(Long Answer Type Questions)

(Marks: 2X6=12)

13. (a) Derive the partial differential equation by eliminating arbitrary function ϕ from the equation $\phi(u,v) = 0$ where u and v are functions of x , y and z .(b) Write short note on surfaces orthogonal to a given system of surfaces.

14. Find the characteristic equations of the pde $z = p^2 - q^2$ and hence determine the characteristic curve . Also find the integral surface which passes the parabola $y = 0, x^2 + 4z = 0$.

15. (i) Find the canonical form of the pde $3u_{xx} + 10u_{xy} + 3u_{yy} = 0$ and solve it ; (Ii) Find the canonical form of the pde $u_{xx} + x^2 u_{yy} = 0$.