24|3|2018

Roll No..................

Date of Exam..................

Department of Mathematics, SOPS, Doon University Dehradun

Mid-Semester Examination 2017-18

M.Sc. Mathematics-I (Second Semester)

Course Title & Course Code: Complex Analysis (MAC-452)

Time: 02 Hour **Total Marks: 30**

Note: (i) Attempt ALL the questions. (ii) Do neat and clean work.

Section A

Attempt ALL: **(2x3=6)**

1. Suppose that $z = x + iy, prove\ that\ |x| + |y| \leq \sqrt{2}|x + iy|$
2. Show that $f(z) = Re(z)$ is not differentiable at any z.
3. If $f(z)$ is analytic at z_0, prove that it must be continuous at z_0.

Section B

Attempt ALL: **(4x3=12)**

1. Prove that a (i) necessary and (ii) sufficient condition that $w = f(z) = u(x, y) + iv(x, y)$ be analytic in a region R is that the Cauchy-Riemann equation $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}; \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$ are satisfied in R where it is supposed that these partial derivatives are continuous in R.
2. In a two dimensional fluid flow, the stream function is $\psi = -\frac{y}{x^2+y^2}$, find the velocity potential function ϕ.
3. (i) Determine whether $|z|^2$ a derivative anywhere has.
   (ii) Find the developments of $\frac{1}{((z-1)(z-2)}$ in powers of z according to the point in z-plane. Expand the function in Taylor's series about z=2 and indicate the circle of convergence.
4. Verify Cauchy's theorem for the integral of z^3taken over the boundary of the rectangle with the vertices $-1, 1, 1 + I, -1 + i$.

Section C

Attempt ALL: **(3x4=12)**

1. (i) State and prove Cauchy's integral formula for the highest order derivative of any analytic function.

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(ii) If C is a unit circle about the origin, described in positive sense, show that $\int_C \frac{e^{-z}}{z^2} dz = -2\pi i \text{ and } \int_C \left(\frac{\sin z}{z}\right) dz = 0.$

2. Prove that function $f(z) = u + iv$, where $f(z) = \frac{x^3(1+i)-y^3(1-i)}{x^2+y^2}, z \neq 0 \text{ and } f(0) = 0$ is continuous and that Cauchy-Riemann equations are satisfied at the origin yet $f'(z)$ does not exist at $z = 0$.

3. (i) If $f(z)$ is an analytic function with constant modulus, then it is constant.

(ii) If $f(z) = u + iv$ is an analytic function of z, and $u - v = \frac{\cos x + \sin x - e^{-y}}{2\cos x - e^y - e^{-y}}$ find $f(z)$ subject to the condition $f\left(\frac{\pi}{2}\right) = 0$.

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