26/3/2018

DOON UNIVERSITY, DEHRADUN
Mid Semester Examination, 2017-2018
School of Physical Sciences
B.Sc.(Mathematics)IV-Semester
Course: MAC-252: Riemann Integration and Series of Functions

Time Allowed: 2 Hours | **Maximum Marks: 30**

Note:

Attempt all questions from Section A, any four questions from Section B and any two questions from Section C.

Section: A ($1.5 \times 4 = 6$ Marks)

(1) With proper notations, define the terms, upper Darboux sum, lower Darboux sum and the Riemann sum.

(2) Show that the Dirichlet function defined on $[0,1]$ is not integrable.

(3) Find two values of c guaranteed by the mean value theorem for integrals for the functions $f(x) = \sin(x)$ on $[0, \pi]$.

(4) Evaluate

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{n+ck}, c > 0.$$

(5) Show that for the logarithm function $L(x)$, $\frac{x-1}{x} \leq L(x) < x - 1$, for $x > 0$.

Section: B ($3 \times 4 = 12$ Marks)

(1) Apply the Darboux approach to compute the integral $\int_0^1 e^x \, \mathrm{d}x$.

(2) Prove that every monotone function f on $[a,b]$ is integrable.

(3) Let f and g be two bounded functions defined on $[a,b]$. Prove that for any partition P, we have $L(P,f) + L(P,g) \leq L(P, f+g)$.

(4) Find an standard partition P of $[0,1]$ such that $U(P,f) - L(P,f) < 0.005$, where $f(x) = 10x$.

(5) Let f be continuous on $[0,1]$, then using the generalized mean value theorem prove that

$$\lim_{n\to\infty} \int_0^1 \frac{nf(x)}{1+n^2x^2} \, \mathrm{d}x = \frac{\pi}{2} f(0).$$

Section: C ($6 \times 2 = 12$ Marks)

Prove following prepositions:

(1) Let P, Q be the partitons of $[a,b]$ then
   (i) $L(P,f) \leq L(P \cup Q, f)$,
   (ii) $U(P \cup Q, f) \leq U(P,f)$.

(2) If f is a bounded and integrable on $[a,b]$ then for each $\epsilon > 0$ there is a partition P of $[a,b]$ such that $U(P,f) - L(P,f) < \epsilon$.

(3) (i) Let f be integrable on $[a,b]$ and G be such that $G(x) = \int_a^x f(x), \mathrm{d}x, x \in [a,b]$, then show that $G(x)$ is continuous on $[a,b]$.
   (ii) $E(ax) = (E(x))^a$, $a \in \mathbb{Q}$.