Roll No.....
Date of Exam. 22-3-2018

Department of Mathematics, SOPS, Doon University Dehradun
Mid-Semester Examination 2017-18
Integrated M.Sc. Mathematics-I (VI Semester)
Course Title & Course Code: Metric Space & Complex Analysis (MAC-351)

Time: 02 Hours **Total Marks: 30**

Note: (i) Attempt ALL the questions. (ii) Do neat and clean work.

Section A

Attempt ALL: **(2x3=6)**

1. Show that $f(z) = Re(z)$ is not differentiable at any z.
2. Describe and plot the curve represented by $z(t) = t^2 + 2it, \quad 0 \leq t < \infty$
3. Describe the interior point of a metric space. Let A be a set of a metric space then prove that A is open$iff\ A = A^o$.

Section B

Attempt ALL: **(4x3=12)**

1. Show that for any two points x and y of a metric space there exist disjoint open balls such that one is centered at x and other at y.
2. Define a metric space. Show that in a metric space $(X, d)\ if\ d^*(x, y) = min\{1, d(x, y)\}$ then d* is also a metric for X.
3. Let A and B be two separated subsets of a metric space (X, d), then prove that (i) if $A \cup B$ is closed then A and B are closed. (ii) if $A \cup B$ is open then A and B are open.
4. If $z_1, and\ z_2$ are two non-zero complex numbers s.t.$|z_1 + z_2| = |z_1| + |z_2|$, then find $\arg(z_1) - \arg(z_2)$.

Section C

Attempt ALL: **(3x4=12)**

1. Determine the region of the z-plane for which$|z - 1| + |z + 1| \leq 3$.
2. Prove that the area of a triangle whose vertices and the points z_1, z_2, z_3 on the argand diagram is $\sum\left\{\frac{(z_2 - z_3)|z_1|^2}{4i\, z_1}\right\}$, show that the triangle is equilateral if ${z_1}^2 + {z_2}^2 + {z_3}^2 = z_1 z_2 + z_2 z_3 + z_3 z_1$
3. Let $X = R^2$, and $x, y \in R^2$ defined $d(x, y) = d\big((x_1, x_2), (y_1, y_2)\big) = \begin{cases} |x_1 - y_1| & if x_2 = y_2 \\ |x_1| + |x_2 - y_2| + |y_1|, & ow \end{cases}$
   Show that (X, d) is a metric space.

****