Roll No:__________

28-3-2018

DOON UNIVERSITY, DEHRADUN

Department of Mathematics, School of Physical Sciences

Mid Semester Examination, Even Semester 2017-18

Class : Int.M.Sc.Mathematics	**Semester : VI**
Course: Ring Theory and Linear Algebra II	**Course Code: MAC-303**
Time Allowed : 2 Hours	**Max Marks : 30**

Note: Attempt all **Six** questions in Section A. Each question carries **1** marks.
Attempt any **Four** questions in Section B. Each question carries **3** marks.
Attempt any **Three** questions in Section C. Each question carries **4** marks.

Section: A

(Very Short Answer Type Questions)

Attempt all SiX questions. **[6×1 = 6 Marks]**

1. The units in ring $Z[i] = \{m + ni| \;\; m, n \in \mathbb{Z}\}$ are
(a)± 1 (b) $\pm i$ (c) $\pm 1, \pm i$ (d) $+1, +i$.

2. The gcd in $Z[i]$ of 2 and $3 + 5i$ is
(a)$1 + i$ (b) $1 - i$ (c) $\frac{1}{2}(1 + i)$ (d) $1 + 2i$.

3. Let R be ring of integers modulo 4. Let $f(x) = 1 + 2x^2$ and $g(x) = 3 + x + 2x^3$ be in $R[x]$, then the $deg(f(x) \cdot g(x))$ is (a)1 (b) 6 (c) 2 (d) 5.

4. The number of elements in the field $\langle \frac{Z_{11}[x]}{<x^2+1>} \rangle$ are
(a)11 (b) 12 (c) 144 (d) 121.

5. Which of the following are/is primitive polynomial
(a)$8x^3 + 6x + 3$ (b) $8x^3 + 6x^2 + 2$ (c) $9x^3 + 9x^2 + 3$(d) $6x^2 + 12x$.

6. If U is a subset of inner product space V. Then the orthogonal complement of U is

   (a) $U^{\perp} = \{v \in V| < v, u >= 0 \;\text{ for some }\; u \in U\}$

   (b) $U^{\perp} = \{v \in V| < v, u >= 0 \;\text{ for every }\; u \in U\}$

   (c) $U^{\perp} = \{v \in V| < v, u >= 0 \;\text{ for particular value of }\; u \in U\}$

   (d) $U^{\perp} = \{v \in V| < v, u >\neq 0 \;\text{ for every }\; u \in U\}$.

Section B

(Short Answer Type Questions)

Attempt any Four questions. **[4×3 = 12 Marks]**

7. Suppose u and v are two elements of an inner product space V. Then show that
$\| u + v \| \leq \| u \| + \| v \|$, the equality holds *iff* u, v are non-negative multiple of other.

8. Prove that the set $\{(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}), (\frac{1}{2}, \frac{1}{2}, \frac{-1}{2}, \frac{-1}{2}), (\frac{1}{2}, \frac{-1}{2}, \frac{-1}{2}, \frac{1}{2}), (\frac{-1}{2}, \frac{1}{2}, \frac{-1}{2}, \frac{1}{2})\}$ form an orthogonal basis of R^4.

9. Discuss the irreducibility criteria for $x^4 + 1$ over Q.

10. If F is a field then show that $F[x]$ is an Euclidean domain.

11. Show that the ideal $< x + 2 >$ is a maximal ideal of $Q[x]$ and hence $Q[x]/ < x + 2 >$ is a field.

Section C

(Long Answer Type Questions)

Attempt any Three questions. **[3×4 = 12 Marks]**

12. State and prove the factor theorem.
13. show that $A = \{xf(x) + 2g(x) | f(x), g(x) \in Z[x]\}$ is not a principal ideal of $Z[x]$ and so, $Z[x]$ is not a principal ideal domain.
14. Prove that $Z[\sqrt{-3}]$ is not a Unique Factorization domain, Z being the ring of integers.
15. Find an orthogonal basis of $P_2(R)$, where the inner product is given by $< p, q >= \int_{-1}^{1} p(x)q(x)dx$.