Roll No:__________

27/3/2018

DOON UNIVERSITY, DEHRADUN

Department of Mathematics, School of Physical Sciences

Mid Semester Examination, Even Semester 2017-18

Class : Int.M.Sc.Mathematics — **Semester : IV**

Course: Ring Theory and Linear Algebra I — **Course Code: MAC-253**

Time Allowed : 2 Hours — **Max Marks : 30**

Note: Attempt all **Six** questions in Section A. Each question carries **1** marks.
Attempt any **Four** questions in Section B. Each question carries **3** marks.
Attempt any **Three** questions in Section C. Each question carries **4** marks.

Section: A

(Very Short Answer Type Questions)

Attempt all Six questions. **[6×1 = 6 Marks]**

1. The ring Z_6 is:
   (a)a field (b) an integral domain (c) a division ring (d) not an integral domain.

2. The set $R = \left\{ \begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix} : a, b \in Z \right\}$ is a ring
   (a)with unity (b) without unity (c) commutative (d) commutative with unity.

3. The characteristic of an integral domain is/are
   (a) zero (b) an integer (c) zero or prime (d) always zero.

4. Number of zero divisors in Z_6 are
   (a) 2 (b) 3 (c) 4 (d) 1.

5. Let R be a ring and $a \in R$ is a nilpotent element of R, then which one is true
   (a) ab is nilpotent for each $b \in R$
   (b) ab is not nilpotent for each $b \in R$
   (c) $a + b$ is nilpotent for each $b \in R$
   (d) $a + b$ is nilpotent for some $b \in R$

6. Which of the following is true
   (a) Z and Z_2 both are fields
   (b) Z and Z_2 both are integral domain but not field
   (c) Z and Z_2 both are integral domain but Z_2 is field only
   (d) Z and Z_2 both are integral domain but Z_2 is not field.

Section B

(Short Answer Type Questions)

Attempt any Four questions. **[4×3 = 12 Marks]**

7. Prove or disprove that subring of a non-commutative ring is non-commutative.

8. Prove that the characteristic of any integral domain R is either zero or prime.

9. Let R be the ring of 3×3 matrices over reals. show that the set $S = \left\{ \begin{pmatrix} x & x & x \\ x & x & x \\ x & x & x \end{pmatrix} : x \in R \right\}$ is a subring of R and has unity different from the unity of R.

10. Show that the ring Z_p of integers modulo p is a field *iff* p is prime.

11. Is union of two ideal of a ring R again an ideal ? If no give an example.

Section C

(Long Answer Type Questions)

Attempt any Three questions. **[3×4 = 12 Marks]**

12. Show that the set $Q = \{a_0 + a_1 i + a_2 j + a_3 k | a_0, a_1, a_2, a_3 \in R\}$, where $i^2 = j^2 = k^2 = ijk = -1$, $ij = -ji = k$, $jk = -kj = i$, $ki = -ik = j$ is a division ring which is not a field.

13. Define an ideal of a ring R and show that the intersection of any two ideal is again an ideal of R.

14. Show that the center of a division ring is a field.

15. If in a ring R with unity satisfying $(xy)^2 = x^2 y^2 \forall x, y \in R$, prove that R is commutative.