Roll No: 24/3/2018

DOON UNIVERSITY, DEHRADUN
Department of Mathematics, School of Physical Sciences
Mid Semester Examination, Even Semester 2017-18

Class : Int. M.Sc. Mathematics — Semester : II
Course: Real Analysis — Course Code: MAC-151
Time Allowed : 2 Hours — Max Marks : 30

Note: Attempt all **Four** questions in Section A. Each question carries **1.5** marks.
Attempt any **Four** questions in Section B. Each question carries **3** marks.
Attempt any **Two** questions in Section C. Each question carries **6** marks.

Section: A
(Short Answer Type Questions)

Attempt all Four questions. **[4×1.5 = 6 Marks]**

1. Define the following terms:
   (a) Bounded sequence (b) Dense set (c) Compact set.
2. Find the derived set of the following sets:
   (a) $\{x : 0 < x < 1, \quad x \text{ is an irrational number}\}$
   (b) $\{\frac{1}{3^m} + \frac{1}{5^n} : n, m \in N\}$
   (c) $\{\frac{n}{n+1} : n \in N\}$.
3. Prove that A^o is the largest open set contained in A.
4. Prove that the set $N \times N$ is countable.

Section: B
(Short Answer Type Questions)

Attempt any Four questions. **[4×3 = 12 Marks]**

5. Prove that between two different rational numbers, there lie an infinite number of rational numbers.
6. If A and B are bounded subsets of R then prove that the set $A + B = \{x + y : x \in A \text{ and } y \in B\}$ is also bounded and $Sup(A + B) = Sup(A) + Sup(B)$.
7. Prove that N^c and Z^c are open sets.
8. Prove that for any set A, $A'' \subset A'$.
9. (a) Prove that the union of a finite family of compact sets is compact.
   (b) Prove that the intersection of an arbitrary family of compact sets with at least one element in common is compact.

Section: C
(Long Answer Type Questions)

Attempt any Two questions. **[2×6 = 12 Marks]**

10. Prove that the set of rational numbers is not order complete.
11. Prove that
    (a) For any set, $\overline{A}$ is a closed set.
    (b) A sequence $\{a_n\}$ is bounded *iff* $\exists$ a positive real number M such that $|a_n| \leq M, \quad \forall n \in N$.
12. State and prove Bolzano-Weierstrass theorem for limit point of a set.