19-12-2017

DOON UNIVERSITY, DEHRADUN

End Semester Examination, Odd Semester, 2017-18

Department of Mathematics, School of Physical Sciences

Class: M.Sc. Mathematics **Semester: III**

Course: Integral Equations and Calculus of Variations **Course Code: MAC-504**

Time Allowed: 3Hours *Maximum Marks: 100*

Note: Attempt all Six questions in Section A. Each question carries 5 marks.
Attempt any Seven questions in Section B. Each question carries 10 marks.

SECTION: A
(Short Answer Type Questions)

(Marks:6X5=30)

1. Solve the following Fredholm integral equation using resolvent kernel method.
   $u(x) = \cos 2x + \int_0^{2\pi} \sin x \cos t\, u(t)dt$
2. Solve the following Volterra integral equation by using Laplace transform.
   $u(x) = e^{-x} + \int_0^x e^{-(x-t)} \sin(x-t)u(t)\, dt$
3. Solve the following integral equation using separable kernel method.
   $u(x) = 1 + \lambda \int_0^{2\pi} \sin(x+t)\; u(t)\, dt$
4. Reduce the following boundary value problem into an integral equation
   $y'' + \lambda y = 0,\; y(0) = 1,\; y(l) = 0.$
5. Find the extremum of the function $\int_0^1 (e^y + xy')dx$, with conditions
   $y(0) = 0,\; y(1) = \alpha.$
6. Find the extremals of the following function
   $\int_0^{\frac{\pi}{4}} (2z - 4y^2 + y'^2 - z'^2)dx,\;\; y(0) = z(0) = 0,\; y\left(\frac{\pi}{4}\right) = z\left(\frac{\pi}{4}\right) = 1.$

SECTION: B
(Long Answer Type Questions)

(Marks: 7X10=70)

7. Convert $y'' - 3y' + 2y = 4\sin x$ with initial conditions $y(0) = 1,\; y'(0) = -2$ into Volterra integral equation of the second kind. Conversely, derive the original differential equation with initial conditions from the integral equation obtained.
8. Determine the eigenvalues and eigenfunctions of the homogeneous integral equation
   $u(x) = \lambda \int_0^1 K(x,t)u(t)dt$, where
   $$K(x,t) = \begin{cases} (1+x)t, & 0 \le x \le t \\ (1+t)x, & t \le x \le 1. \end{cases}$$

9. Solve the Abel integral equation. Using the result, find the solution of the following singular integral equation.

$$x^2 = \int_0^x \frac{y(t)}{(x-t)^{1/2}}\, dt.$$

10. Using Hilbert-Schmidt theorem, solve the following symmetric integral equation:

$$u(x) = 1 + \lambda \int_0^1 \cos(x+t)\, u(t)\, dt.$$

11. Use the method of successive approximations, solve the integral equation

$$u(x) = \frac{x^3}{3} - 2x - \int_0^x u(t)\, dt\ , \text{ taking } \ u_0(x) = x^2.$$

12. Find the Neumann series for the solution of the integral equation

$$u(x) = 1 + x + \lambda \int_0^x (x-t)\, u(t)\, dt.$$

13. Find the extremals of the following function

$$\int_0^1 \left(y'^2 + z'^2 - 4xz' - 4z\right) dx, \quad y(0) = z(0) = 0, \ y(1) = z(1) = 1 \ \text{ subject to the condition}$$
$$\int_0^1 (y'^2 - xy' - z'^2)\, dx = 2.$$

14. Find the broken line extremals of the function

$$\int_0^4 (y'-1)^2 (y'+1)^2 dx, \quad y(0) = 0, \ y(4) = 2.$$