7-12-17

DOON UNIVERSITY, DEHRADUN

End Semester Examination, Odd Semester, 2017-18

Department of Mathematics, School of Physical Sciences

Class: M.Sc. Mathematics **Semester: III**

Course: Non-Linear Programming Problem **Course Code: MAC-501**

Time Allowed: 3Hours *Maximum Marks: 100*

Note: **Attempt all Four questions in Section A. Each question carries 5 marks.**
Attempt any Four questions in Section B. Each question carries 10 marks.
Attempt any Two questions in Section C. Each question carries 20 marks.

SECTION: A
(Very Short Answer Type Questions)

(Marks:4X5=20)

1. Solve the following linear programming problem(LPP) using simplex method
   Maximize $Z = x_1 + 2x_2 + 3x_3$
   Subjected to : $x_1 + 2x_2 + 3x_3 \leq 10;\quad x_1 + x_2 \leq 5;\quad x_1, x_2, x_3 \geq 0.$
2. Solve the following LPP using dual simplex method
   Minimize $Z = x_1 + x_2$
   Subjected to : $2x_1 + x_2 \geq 4;\quad x_1 + 7x_2 \geq 7;\quad x_1, x_2 \geq 0.$
3. Use graphical method to solve the following non-linear programming problem:
   Maximize $f(X) = (x_1 - 2)^2 + (x_2 - 3)^2$
   Subjected to: $3x_1 + 2x_2 \geq 6;\quad -x_1 + x_2 \leq 3;\quad x_1 \leq 2.$
4. Let $S \subseteq R^n$ be a convex Set and $f: S \to R$, then prove that f is a convex function on S if and only if its epigraph E_f is a convex set.

SECTION: B
(Short Answer Type Questions)

(Marks: 4X10=40)

5. Solve the following non-linear programming problem using the method of Lagrangian multipliers.
   Maximize $f(X) = 6x_1 + 8x_2 - x_1^2 - 2x_2^2$
   Subjected to: $4x_1 + 3x_2 = 16;\quad 3x_1 + 5x_2 = 15;\quad x_1, x_2 \geq 0.$
6. Use Kuhn-Tucker conditions to solve the following non-linear programming problem:
   Minimize $f(X) = 2x_1 + 3x_2 - x_1^2 - 2x_2^2$
   Subjected to: $x_1 + 3x_2 \leq 6;\quad 5x_1 + 2x_2 \leq 10;\quad x_1, x_2 \geq 0.$

7. Let $f: S \to R$ be differentiable function on an open convex subset S of R^n. Prove that f is convex function if and only if $f(X_1) - f(X_2) \geq (X_1 - X_2)^T \nabla f(X_2), \quad \forall X_1, X_2 \in S$.

8. Use the method of separable convex programming for solving the following non-linear programming problem:

Maximize $f(X) = x_1^2 - 5x_1 + x_2^2 - 5x_2 - x_3$
Subjected to: $x_1 + x_2 + x_3 \leq 4$; $x_1^2 - x_2 \leq 3$; $x_1, x_2, x_3 \geq 0$.

9. Use dynamic programming to solve the following non-linear programming problem:

Minimize $Z = x_1^2 + x_2^2 + x_3^2$
Subjected to: $x_1 + x_2 + x_3 \geq 15$; $x_1, x_2, x_3 \geq 0$.

SECTION: C
(Long Answer Type Questions)

(Marks: 2X20=40)

10. (a) State and prove the necessary an sufficient condition for the minimum or maximum of an unconstrained non-linear programming problem of n variables. **(12+8)**

(b) Let $S \subseteq R^n$ be a non-empty convex set and $f: S \to R$ be twice differentiable on S. Then prove that f is a convex function on S if and only if the Hessian matrix $\nabla^2 f(X)$ is positive semi-definite $\forall\, X \in S$..

11. (a) Use Wolfe's method in solving the following quadratic programming problem **(12+8)**

Maximize $f(X) = 2x_1 + 3x_2 - 2x_1^2$
Subjected to: $x_1 + 4x_2 \leq 4$; $x_1 + x_2 \leq 2$; $x_1, x_2 \geq 0$.

(b) Solve the following 3×3 game by LPP.

		Player B	
	1	-1	3
Player A	3	5	-3
	6	2	-2

12. (a) Solve the following geometric programming problem **(12+8)**

Minimize $f(X) = 5x_1^{-1}x_2^{-1}x_3^{-1} + 5x_2x_3$
Subjected to: $2x_1x_3 + x_1x_2 = 4$; $x_1, x_2, x_3 \geq 0$.

(b) Use Two-Phase method to solve the following LPP:
Maximize $Z = 3x_1 - x_2$
Subjected to: $2x_1 + x_2 \geq 2$; $x_1 + 3x_2 \leq 2$; $x_2 \leq 4$; $x_1, x_2 \geq 0$.