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DOON UNIVERSITY, DEHRADUN
End Semester Examination, First Semester, 2017
School of Physical Sciences
M.Sc.(Mathematics)
Course: MAC-406: Linear Algebra

Time Allowed: 2 Hours | Maximum Marks: 50

Note:

1. **Attempt any nine Questions from Sections A.**
2. **Attempt any four Questions from Sections B.**
3. **Attempt any two Questions from Sections C.**

SECTION: A

($9 \times 2 = 18$ Marks)

1. Let T be a linear transformation from R^7 onto a 3-dimensional subspace of R^5.Then dim Ker(T) is ...
2. A linear operator T is invertible iff ker(T) = ...
3. If V is an inner product space and if $\{w_1, w_2, ..., w_n\}$ is an orthonormal set in V, then $\sum_{i=1}^{n} |< w_i, v >|^2 \leq ....$
4. If S_1 and S_2 are subsets of an inner product space V, then $S_1 \subseteq S_2 \implies ....$
5. A matrix A is diagonalizable if there an ... matrix P such that ....
6. If $V = C(R)$ and $W = R(R)$. Then dim $\frac{V}{W} =$.
7. If α and β are vectors in a Unitary space, then x and y are ....
8. The minimal polynomial for the matrix
$$M = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$
is
9. If T is a diagonalizable linear operator, then the minimal polynomial for T is a ....
10. If in an inner product space $||x + y|| = ||x|| + ||y||$, then x and y are ....

SECTION: B

($5 \times 4 = 20$ Marks)

1. Let V be the vector space of all real-valued continuous fuctions.Show that the linear operator $T : V \to V$ defined as $(Tf)(x) = \int_0^x f(t)dt$ has no eigenvalues.
2. State and prove Pythagoras theorem.
3. Let T be a linear transformation from R^3 into R^2 and let U be a linear transformation from R^2 into R^3.Prove that the linear transformation UT is not invertible.
4. Show that an orthogonal set of nonzero vectors in an inner product space V is linearly independent.

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5. Let $R_4[x] = \{a_0 + a_1x + a_2x^2 + a_3x^3 | a_i \in R\}$.Define $T : R_4[x] \to R_4[x]$ as $(T(f)) = \frac{df(x)}{dx}$ for all $f(x) \in R_4[x]$.Let $\beta = \{1, x, x^2, x^3\}$ be an ordered basis of $R_4[x]$. Find $[T]_\beta$.

SECTION: C

(2 × 6 = 12 Marks)

1. Let V be real functions satisfying $\frac{d^2y}{dx^2} + 9y = 0$ with inner product is defined by $< y, z >= \int_0^\pi yzdx$.Find an orthonormal basis of V.
2. Determine all possible jordan Canonical forms for a linear operator $T : V \to V$ whose charactristic poynomial $(t-2)^3(t-5)^2$.In each case find the minimal polynomial.
3. Let $A : R^4 \to R^3$ where

$$M = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 3 & 5 & -2 \\ 3 & 8 & 13 & -3 \end{bmatrix}$$

.Find a basis and the dimension of image of A and the Kernel of A.