



M.Sc. (Mathematics)-I (First Semester)
End Semester Theory Examination 2017-18
Department of Mathematics, SOPS, Doon University Dehradun
Core Course, Mathematical Modelling (MAC-405)

Time: 03 Hour

Total Marks: 100

Note: (i) Attempt ALL the questions. (ii) Do neat and clean work.

Attempt any TEN:

(10x10=100)

1. Discuss various ways to study a system. Draw a flow chart explaining various steps involved in mathematical modelling.
2. (a) Explain the modelling for the motion of a satellite.
(b) Discuss and prove Kepler's law of planetary motion.
3. A particle moves in a plane with an acceleration which is always directed to a fixed point O in the plane; to obtain the differential equation of the path.
4. Write a short note on: (i) Diffusion Glucose or medicine in the blood stream (ii) Competition model. (iii) Inventory control model.
5. Show that the motion of a projectile is parabolic and also show that it will attain the maximum height $\frac{v^2 \sin^2 \alpha}{2g}$ at time $\frac{v \sin \alpha}{g}$.
6. (a) Draw some trajectories for the model $\frac{dx}{dt} = x(1 - 0.1y)$; $\frac{dy}{dt} = -y(1 - 0.1x)$
(b) In an archeological wooden specimen, only 25% of original radio carbon-12 is present. When was it made?
7. (a) Explain the Mathematical Modelling of diffusion of glucose or a medicine in the blood stream.
(b) Solve SIS model when β is a known function of t.
8. A population is decaying exponentially. Can this decay be stopped or reversed by an immigration at a large constant rate into the population?
9. Initially, if there are n susceptibles and a infective, find expressions for s(t) and I(t) and discuss the variation of both with a.
10. Assuming that a particle projected vertically upwards from the surface of the earth moves in vacuum under a force ga^2/x^2 directed towards the centre of earth, where x is the distance of the particle from the centre of the earth, find the initial velocity of projection so that the particle never return to earth.
11. A lake of constant volume V contains at time t an amount Q(t) of pollutant evenly distributed throughout the lake. Suppose water containing concentration k of pollutant enters the lake at a rate re and water leaves the lake at the same rate r and water leaves the lake at the same rate. Suppose pollutants are also added to the lake at a constant rate P. Discuss the mathematical modelling. If initial concentration of the lake is c_0 , find c(t).