16/12/17

DOON UNIVERSITY, DEHRADUN

End Semester Examination, Odd Semester, 2017-18

Department of Mathematics, School of Physical Sciences

Class: M.Sc. Mathematics | **Course: Numerical Analysis**

Semester: I | **Course Code: MAC-404**

Time Allowed: 3Hours | *Maximum Marks: 60*

Note: **Attempt all Four questions in Section A. Each question carries 4 marks.**
Attempt any Four questions in Section B. Each question carries 5 marks.
Attempt any Three questions in Section C. Each question carries 8 marks.

SECTION: A
(Very Short Answer Type Questions)

(Marks:4X4=16)

1. Define the following terms:
   (a) Zero stability and root condition for a linear multi-step method
   (b) Order of Convergence of a method to find root of the equation $f(x) = 0$
   (c) Nonlinear boundary value problem of second order with Dirichlet and non-Dirichlet boundary condition.
   (d) Quadratic Spline
2. Derive the formula for Newton Raphson method. Hence, find the root of the equation $x\log_{10}x = 1.2$ correct to five decimal places in the interval $(1, 3)$.
3. Use the numbers $x_0 = 2$, $x_1 = 2.75$, and $x_2 = 4$ to find the second Lagrange interpolating polynomial for $f(x) = \frac{1}{x}$. Hence find the value of $f(3.25)$ using interpolating polynomial.
4. Solve by Taylor's series method the equation $y' = \log(xy)$ for $y(1.1)$ and $y(1.2)$ given that $y(1) = 2$.

SECTION: B
(Short Answer Type Questions)

(Marks: 4X5=20)

5. Obtain the piecewise cubic interpolating polynomial for the function $f(x)$ defined by the given data. Also interpolate it at $x = 0$ and $x = 3$.

x	-4	-3	-1	1	2	4	5
$f(x)$	-108	-255	-303	-303	-300	-108	225

6. Fit the following four points by the cubic splines:

x	1	2	3	4
$f(x)$	1	5	11	8

   Use the end conditions $f''(1) = f''(4) = 0$. Hence find an estimate of $f(1.5)$.
7. Given $y' = y\cos x$, $y(0) = 1$. Compute $y(0.2)$ and $y(0.4)$ with $h = 0.2$ using Improved Euler's method.

8. Derive Milne's Predictor corrector formula to find approximate solution of an initial value problem.

9. Solve the following boundary-value problem using finite difference method.

$$y'' = (2 + 4x^2)y, \quad y(0) = 1, \quad y(1) = e$$

SECTION: C

(Long Answer Type Questions)

(Marks: 3X8=24)

10. Derive formula for Hermite interpolating polynomial $p(x)$ for the function $f(x)$ with interpolating conditions $p(x_i) = f(x_i)$ and $p'(x_i) = f'(x_i)$, $i = 0,1, \ldots n$. Hence, find the value of $f(0.5)$ from the following data:

x	-1	0	1
$f(x)$	1	1	3
$f'(x)$	-5	1	7

11. Using Runge-Kutta method of order four, calculate $y(0.1), y(0.2), y(0.3), y(0.4)$ given that $y' = x - y^2$, $y(0) = 1$. Taking these values as starting values, find $y(0.5)$correct to three decimal places using Adams–Bashforth and Adams–Moulton methods as predictor-corrector respectively.

12. Use Newton's method with $x^{(0)} = 0$ to compute $x^{(3)}$ for the following nonlinear system.

$$5x_1^2 - x_2^2 = 0$$

$$x_2 - 0.25(\sin x_1 + \cos x_2) = 0\,.$$

13. Find the interval of absolute stability for the two-step implicit method

$y_{n+1} = 3y_n - 2y_{n-1} + \frac{h}{12}[13f_{n+1} - 20f_n - 5f_{n-1}]$ using

(*a*) Schur's criterion

(*b*) Routh–Hurwitz criterion.