19-12-2017

M.Sc. (Mathematics)-I (First Semester)

End-Semester Theory Examination 2017-18

Department of Mathematics, SOPS, Doon University Dehradun

Core Course: MAC-403, Ordinary Differential Equation

Time Allowed: 3Hours *Maximum Marks:* **60**

Note: Attempt All Questions from Sections A, B, C.

SECTION A

Attempt ALL Questions. **(2x5=10)**

1. Use Wronskian to show that the functions x, x^2, x^3 are independent. Determine the differential equation with these as independent solutions.
2. What do you understand by the Orthogonal trajectory and phase plane?
3. Define Green's function in a proper way.
4. How can you transform an ODE into a system of ODE?
5. What kind of critical point does $my'' + cy' + ky = 0$?

SECTION B

Attempt any FOUR Questions. **(4x5=20)**

1. Discuss various types of stabilities.
2. Define Lipschitz condition.
3. Find the graph and general solution of $y' = Ay = \begin{bmatrix} -3 & 1 \\ 1 & -3 \end{bmatrix}$y, on the basis of Eigenvector available also find which type of node exist.
4. Show that for the problem $\frac{dy}{dx} = y, y(0) = 1$, the constant a in Picards theorem must be smaller than unity.
5. Find the general solution of the given ODE by first converting it to a system $y'' + 2y' - 24y = 0$

SECTION C

Attempt ALL Questions. **(5x6=30)**

1. State and Prove the Existence and Uniqueness theorem.
2. Using Green's function, solve the boundary value problem $y'' - y = x. y(0) = y(1) = 0$.
3. Solve the differential equation $\frac{dy}{dx} = x - y$, with the condition y=1 when x=0 and show that the sequence of approximations given by Picards method tend to the exact solution as a limit.
4. Find the solution of the linear system of differential equation $y' = \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix} y$, also discuss the type of critical point.
5. Tank T_1 contains initially 200gal of water in which 160lb of salt are dissolved. Tank T_2 contains initially 100gal of pure water. Liquid is pumped through the system as indicated, and the mixtures are kept uniform by stirring. Find the amounts of salt$y_1(t)$ and $y_2(t)$ in $T_1 and$ T_2, respectively.

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