11-12-17

DOON UNIVERSITY, DEHRADUN

End Semester Examination, First Semester, 2017

M. Sc. Mathematics

Course: MAC-402: Topology

Time Allowed: 3 Hours **Maximum Marks: 100**

Note: *Attempt all questions from section A, any four questions from section B and any two questions from section C.*

In all of the problems, it will be understood that all the sets are subsets of some topological space (X, T).

Section: A ($4\times 5 = 20$ Marks)

Prove following propositions:

(1) A^{o} is the largest open subset of A.

(2) If A is closed, then $A' \subset A$.

(3) (a) Find the closure of the set $\{2, \frac{3}{2}, \frac{4}{3}, \frac{5}{4}, ...\}$ with respect to the usual topology U on $\mathbb{R}$.

(b) Separated sets are disjoint but converse is not true.

(4) Continuous image of a compact set is compact.

(5) (a) The real line $(\mathbb{R}, U)$ is separable.

(b) The constant map $f : X \longrightarrow Y$, defined by $f(x) = c$ is continuous.

Section: B ($10\times 4 = 40$ Marks)

(1) Prove the following propositions:

(i) $A^{o} = (\overline{A^{c}})^{c}$,

(ii) $\overline{A} = ((A^{c})^{o})^{c}$.

(2) A map $f : X \longrightarrow Y$, (X, T) and (Y, V) topological spaces, is continuous iff $f^{-1}(H)$ is $T-$open for every $V-$open set H.

(3) Every compact subset of a Hausdorff space is closed.

(4) (a) Show that the set $(0, 1)$ is not compact in real line.

(b) Show that the map $f : \mathbb{R} \longrightarrow \mathbb{R}$ given by $f(x) = x^2$ is continuous.

(5) Let A and B be separated sets such that $A \cup B$ is closed. Then show that A and B are closed.

Section: C ($20 \times 2 = 40$ Marks)

(1) (i) Let $\mathbb{N}$ be the set of all natural numbers and T a collection of ϕ all subsets of $\mathbb{N}$ of the form $G_m = \{m, m+1, m+2, ...\}$, $m \in \mathbb{N}$. Show that T is a topology.

(ii) $\text{bd}(A) = \overline{A} \cup \overline{A^{c}}$.

(2) (a) Every compact topological space is locally compact but the converse is not true.

(b) Subsets of separated sets are separated.

(3) (i) Show that the map $f : \mathbb{R} \longrightarrow \mathbb{R}$ given by

$$f(x) = \begin{cases} \frac{1}{x}, & x \neq 0 \\ 0, & x = 0. \end{cases}$$

is not $\mathbb{U} - \mathbb{U}$ continuous.

(ii) Prove that

(a) the identity map of any topological space is continuous;

(b) any constant map is continuous;

(c) if $(X, \mathcal{D})$ is the discrete topological space then any map $f : X \to Y$ to another topological space (Y, V) is continuous;

(d) if $(Y, \mathcal{I})$ is the indiscrete topology then any map $f : X \to Y$ from another topological space (X, T) is continuous.