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DOON UNIVERSITY, DEHRADUN

End Semester Examination, First Semester, 2017

School of Physical Sciences

M.Sc.(Mathematics)

Course: MAC-401: Finite Field

Time Allowed: 2 Hours | **Maximum Marks: 50**

Note:

1. **Attempt any nine Questions from Sections A.**
2. **Attempt any four Questions from Sections B.**
3. **Attempt any two Questions from Sections C.**

SECTION: A

($9 \times 2 = 18$ Marks)

1. The degree of $Q(\sqrt[3]{2}, \sqrt[4]{3})$ is....
2. $\frac{Z_3}{<x^3+2x+2>}$ is a field with ... element.
3. The splitting field of $x^3 - 1$ over Q is ....
4. Any finite subgroup of the multiplicative group of a finite field is ....
5. $[GF(p^n) : GF(p^m)] =$ ....(provided m divides n).
6. If $p(x) \in F[x]$ and degree of $p(x)$ is n.Then the splitting field for $p(x)$ over F has degree at most ....
7. Can the cube be tripled? and why?
8. A finite extension of a finite field is a ... extension.
9. The degree of $Q(\sqrt[3]{\pi})$ over Q is ....
10. π and e in R are .... over Q.

SECTION: B

($4 \times 5 = 20$ Marks)

1. Find the degree of minimal splitting field of $x^4 + 2$ over Q.
2. If $a > 0$ is constructible. Then prove that $\sqrt{a}$ is also constructible.
3. Let $x^p - a \in F[x]$ where CharF = p.Then show that either $x^p - a$ is irreducible over F or $x^p - a$ is a p^{th} power of a linear polynomial in F.
4. Discuss the irreducibility of $x^4 + 1$ over rationals.
5. If L is an algebric extension of K and K is an algebric extension of F. Then show that L is an algebric extension of F.

SECTION: C

($2 \times 6 = 12$ Marks)

1. Determine the splitting field of $x^4 + x^2 + 1$ over Q.Also find its degree over Q.
2. If a real number α is constructible. Then prove that α lies in some extension field K of Q such that $[K : Q] = 2^r$ for some non-negative integer r.
3. If a is any algebric number, then show that there exists a positive integer n such that na is an algebric integer.