12-12-17

I M.Sc. Mathematics First Semester
End Semester Examination, 2017-18
Department of Mathematics, SOPS, Doon University, Dehradun
Core Course: MAC-102, Algebra

Time Allowed: 3Hours **Maximum Marks: 100**

Note: Attempt All Questions from Sections A, B,C.

SECTION A

Attempt ALL Questions. **(2x10=20)**

1. A is a 5x5 matrix, all of whose entries are 1, then-
(a)A is not diagonalizable; (b) A is idempotent; (c) A is nilpotent; (d) The minimal polynomial and characteristic polynomial of A are not equal.
2. A is an upper triangular with all diagonal entries zero, then I+A is-
(a)invertible; (b) idempotent; (c) singular; (d) nilpotent
3. $A = \begin{bmatrix} 0 & 1 & a \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}; B = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ $then,$

(a) A and B are similar; (b)A and B are not similar; (c)A and B are nilpotent; (d) A and AB are similar

4. $T: R^4 \rightarrow R^4, defined\ by\ T(e_1) = e_2; T(e_2) = e_3; T(e_3) = 0; T(e_4) = e_3$, then
(a)T is nilpotent; (b) T has at least one non-zero Eigen-value; (c) index of nilpotent is three; (d)T is not nilpotent
5. Let u, v, w be three non-zero vectors which are LI, then- (a) u is a linear combination of v and w; (b) v is the linear combination of u &w; (c) w is the linear combination of u & v; none of these.
6. The sum of the Eigen values of the matrix $\begin{bmatrix} 4 & 7 & 11 \\ 7 & 1 & -21 \\ 11 & -21 & 6 \end{bmatrix}$ is- (a) 4; (b) 23; (c) 11; (d) 12
7. $f: N \rightarrow N; f(x) = 2x\ is$: (a) 1-1 and onto; (b) 1-1 and into; (c) many-one and onto; (d) many-one and into
8. If R be a relation defined as $aRb\ iff\ |a - b| > 0$, then the relation is-
(a) reflexive; (b) symmetric; (c) transitive; (d) symmetric and transitive.
9. Let I be the identity transformation of the finite dimensional vector space V, then the nullity of I is- (a) dimV; (b) 0; (c) 1; (d) dimV-1
10. The Eigen values of a skew-symmetric matrix are –
(a) negative; (b) real; (c) absolute value of 1; (d) purely imaginary or zero.

SECTION B

Attempt any FIVE: **(5x6=30)**

1. Find a 3x3 orthogonal matrix P, whose first two rows are multiples of (1, 2, 3) and (0, -2, 3).
2. Show that A is invertible $iff\ A'$ is invertible.

3. Find the dimension and basis of the general solution of the following system: $2x - 4y + 3z - t + r = 0$; $3x - 6y + 5z - 2t + 4r = 0$; $5x - 10y + 7z - 3t + 18r = 0$.

4. Reduce the matrix $\begin{bmatrix} 1 & 3 & 1 & 3 \\ 2 & 8 & 5 & 10 \\ 1 & 7 & 7 & 11 \\ 3 & 11 & 7 & 15 \end{bmatrix}$ to the canonical form.

5. Find the basis and dimension of the subspace W of R^3 $where\ W = \{(a,b,c): a + b + c = 0\}$

6. (a)Show that every set of three vectors in $R^2(R)$ is LD

(b)Show that the linear transformation $T: R^3 \to R^3 defined\ by\ T(x_1, x_2, x_3) = (2x_1, x_1 - x_2, 5x_1 + 4x_2 + x_3)$ is invertible.

SECTION C

Attempt any FIVE: **(5x10=50)**

1. (a) If in a ring R with unity $(xy)^2 = x^2y^2$ for all $x, y \in R$ then show that R is commutative.

(b) If $S = \{v_1, v_2, \ldots\ldots v_n\}$ is a basis of V then every element of V can be expressed uniquely as a linear combination of $v_1, v_2, \ldots\ldots v_n$.

2. Consider the following system of simultaneous linear equation $x + y + az = 1$; $x + ay + z = 4$; $ax + y + z = b$, for which values of $'a'$ does the system have a unique solution for which pairs of values (a, b).

3. Find the basis and dimension of the image of the mapping $T: R^4 \to R^3$, defined by $T(x\ y\ z\ t) = (x - y + z + t, 2x - 2y + 3z + 4t, 3x - 3y + 4z + 5t)$.

4. Using, Cayley-Hamilton theorem obtain the inverse of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$

5. (a)Define composite numbers and prove that$(n^4 + 4)$ $is\ composite\ if\ n > 1$.

(b)Consider the vector space $M = M_{2,2}$ consisting of all 2x2 matrices, and consider the following four matrices in M: $E_{11} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, E_{12} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, E_{21} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, E_{22} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$, show that the four matrices span M.

6. (a) $if\ a \equiv b(mod n)$. Prove that $gcd\ (a, n) = gcd(b, n)$

(b) Show that for any integer a, $a^3 \equiv 0, 1, 8 (mod\ 9)$.

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