16/12/17

DOON UNIVERSITY, DEHRADUN

End Semester Examination, Odd Semester, 2017-18

Department of Mathematics, School of Physical Sciences

Class: Integrated M.Sc.(PHY, CHE, CS) **Semester: III**

Course: Applications of Algebra **Course Code: MAG-201**

Time Allowed: 3Hours ***Maximum Marks: 100***

Note: **Attempt all Ten questions in Section A. Each question carries 2marks.**
Attempt any Eight questions in Section B. Each question carries 5 marks.
Attempt any Four questions in Section C. Each question carries 10 marks.

SECTION: A
(Very Short Answer Type Questions)

(Marks: 10X2=20)

1. If B is an idempotent matrix, show that $A = I - B$ is also idempotent that $AB = BA = 0$.
2. Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \\ 3 & 6 & 10 \end{bmatrix}$.
3. Classify the following quadratic form as positive definite, positive semi-definite, negative definite, negative semi definite, or indefinite.
   $f(x) = 2x_1^2 + 3x_2^2 - 4x_1x_2$
4. Test the linearly independence or dependence of the vectors $X_1 = (1\ 2\ 3)$ and $X_2 = (3\ \ 6\ 12)$.
5. Prove that the identity element of a group is unique.
6. Show that a cyclic group is necessarily abelian.
7. Determine which of the following permutations are even and odd:

   (i) $f = (1\,2\,3\,4\,5)(1\,2\,3)(4\,5)$ (ii) $f = \begin{pmatrix} 1\ 2\ 3\ 4\ 5 \\ 2\ 3\ 1\ 4\ 5 \end{pmatrix}$

9. Define and explain the following terms:
   (i) Degenerate solution (ii) Convex set
10. Solve the following linear programming problem:
    Max $3x + 2y$
    Subject to : $x + y \leq 2;\ x \geq 0;\ y \geq 0$.

SECTION: B
(Short Answer Type Questions)

(Marks: 8X5=40)

11. Find the minimal polynomial $m(t)$ of $\begin{bmatrix} 2 & 2 & -5 \\ 3 & 7 & -15 \\ 1 & 2 & -4 \end{bmatrix}$.
12. State and prove Cayley Hamilton theorem and verify it for the matrix $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$
13. Write the matrix A of the quadratic form $(x) = 6x_1^2 + 65x_2^2 + 11x_3^2 + 4x_1x_3$. Find the eigenvalues of A and hence determine the nature of definiteness of the quadratic form.
14. Show that the set $S = \{1, 5, 7, 11\}$ is a group with respect to multiplication modulo 12.
15. If H and K are two subgroups of a group G, then $H \cup K$ is a subgroup of G if and only if either $H \subset K$ or $K \subset H$.
16. Prove that the set A_3 of three permutations $(a), (a\ b\ c),\ (a\ c\ b)$ on three symbols a, b, c forms a finite abelian group with respect to the permutation multiplication.

17. Use Graphical method for solving the following non-linear programming problem.

Minimize $f(X) = x_1^2 + x_2^2$

Subject to: $x_1 \leq 10$; $x_1 - x_2^2 - 4 \geq 0$.

18. Use Simplex method to solve the following LPP:

Minimize $Z = 5x_1 + 3x_2$

Subjected to: $4x_1 + 5x_2 \leq 10$; $5x_1 + 2x_2 \leq 10$; $3x_1 + 8x_2 \leq 12$; $x_1, x_2 \geq 0$.

SECTION: C

(Long Answer Type Questions)

(Marks:4X10=40)

19. Let $A = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$.

(a) Find all eigenvalues and corresponding eigenvectors.

(b) Find a nonsingular matrix P such that $D = P^{-1}AP$ is diagonal.

(c) Find A^6 and $f(A)$, where $t^4 - 3t^3 - 6t^2 + 7t + 3$.

20. Find the inverse of the following matrix using Gauss-Jordan method.

$$\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

21. Using Doolittle's method, solve the following system.

$$28x + 4y - z = 32$$
$$x + 3y + 10z = 24$$
$$2x + 17y + 4z = 35$$

22. Use Big-M method to solve the following linear programming problem:

Maximize $Z = 5x_1 + 2x_2$

Subjected to: $3x_1 + 2x_2 \geq 3$; $x_1 + 4x_2 \geq 4$; $x_1 + x_2 \leq 5$; $x_1, x_2 \geq 0$.

23. Prove that a non-empty subset H of a group G is a subgroup of G if and only if $ab^{-1} \in H \ \forall\, a, b \in H$.