15-12-17

DOON UNIVERSITY, DEHRADUN

End Semester Examination, Third Semester, 2017

School of Physical Sciences

Integrated M.Sc.(Mathematics)

Course: MAC-302: Group Theory II

Time Allowed: 2 Hours **Maximum Marks: 50**

Note:

1. **Attempt any nine Questions from Sections A.**
2. **Attempt any four Questions from Sections B.**
3. **Attempt any two Questions from Sections C.**

SECTION: A

($9 \times 2 = 18$ Marks)

1. A group of order 1681 is ....
2. The number of conjugate classes of a non-abelian group of order 27 .....
3. The class equation of S_4 is ....
4. Any group of order 15 is ....
5. If the order of $\frac{G}{Z}$ is 77, then G is ....
6. The number of non isomorphic abelian groups of order 24 is .....
7. If G is a group of order pq such that p and q are distinct prime with $p < q$ and $p \nmid q-1$. Then G is isomorphic to ....
8. If a finite non-abelian group simple group G has a subgroup of index n, then G is isomorphic to a subgroup of ....
9. $cl(a) = \{a\}$ iff $a \in$ ...
10. Let n be the smallest composite integer such that there is a unique group of order n. Then the value of n is ....
11. If p is the smallest prime that divides order of G. Then any subgroup of .... p in G is ... in G.

SECTION: B

($5 \times 4 = 20$ Marks)

1. Prove that if a group G of order 28 has a normal subgroup of order 4, then G is abelian.
2. If M and N are normal subgroups of a group G, then show that $\frac{G}{M \cap N}$ is isomorphic to a subgroup of the direct product of $\frac{G}{M} \times \frac{G}{N}$.
3. If $o(G) = 30$, show that every sylow 3-subgroup and every sylow 5-subgroup of G must be normal in G.
4. If G is a finite abelian group and m is a positive integer such that m divides order of G. then prove that G contains a subgroup of order m.
5. Find all the non-isomorphic abelian groups of order 360.

SECTION: C

($2 \times 6 = 12$ Marks)

1. Let A and B be cyclic groups of orders m and n respectively. Prove that $A \times B$ is cyclic iff m and n are relatively prime.
2. State and prove Cauchy's theorem for finite abelian groups.
3. If G is a group of order pq such that p and q are distinct prime with $p < q$ and $p \nmid q-1$. Then show that G is cyclic.