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DOON UNIVERSITY, DEHRADUN

End Semester Examination, Second Semester, 2017-18

Department of Mathematics, School of Physical Sciences

Class: Integrated M.Sc. Mathematics
Semester: V

Course: Multivariable Calculus
Course Code: MAC-301

Time Allowed: 3 Hours ***Maximum Marks: 100***

Note: Attempt all five questions in Section A. Each question carries 4 marks.
Attempt any four questions in Section B. Each question carries 10 marks.
Attempt any two questions in Section C. Each question carries 20 marks.

SECTION: A
(Very Short Answer Type Questions)

(Marks:5X4=20)

1. Determine divergence and curl of $\hat{r} = (xi + yj + zk)/\sqrt{(x^2+y^2+z^2)}$.

2. Find parametrization of the cone $z = \sqrt{x^2+y^2}$, $0 \le z \le 1$.

3. (i)Write parametric formula for the area of a smooth surface.

(ii)What is the condition for a vector field to be conservative?

4. Find the Jacobean of Cartesian coordinates with respect to cylindrical coordinates.

5. Write spherical coordinates in terms of Cartesian coordinates and vice versa.

SECTION: B
(Short Answer Type Questions)

(Marks: 4X10=40)

6. a. Write short note on maxima and minima test for functions of two variables. And find local extreme values of the function (i) $f(x,y) = xy - x^2 - y^2 - 2x - 2y + 4$; (ii) $g(x,y) = xy$.

b. Using Lagrange's method, find the maximum and minimum values of the function $f(x,y) = 3x + 4y$ on the circle $x^2 + y^2 = 4$.

7. a. State Fubini theorem and verify it by calculating $\iint_R f(x,y)\,dA$ for $f(x,y) = 1 - 6x^2 y$, $0 \le x \le 2$, $-1 \le y \le 1$.

b. Find the volume of the prism whose base is the triangle in the xy-plane bounded by the x-axis and , the line $y = x$ and $x = 1$, whose top lies in the plane $z = f(x,y) = 3 - x - y$.

c. Calculate $\iint_R \frac{\sin x}{x} dA$ where R is the triangle in the xy-plane bounded by the x-axis , the line $y = x$ and the line $x = 1$.

d. Find the area of the region R bounded by $y = x$ and $y = x^2$ in the first quadrant.

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8. a. A thin plate covers the triangular region bounded by the axis and the lines $x=1$ and $y=2x$ in the first quadrant. The plate's density at the point (x,y) is $\delta(x,y)=6x+6y+6$. Find the plate's mass, first moments, center of mass, moments of inertia, and radii of gyration about the coordinate axes.
b. Find the centroid of the region in the first quadrant that is bounded above by the line $y=x$ and below the parabola $y=x^2$.

9. a. Find the centroid of the solid of constant density $\delta=1$ enclosed by the cylinder $x^2+y^2=4$ bounded above by the paraboloid $z=x^2+y^2$ and below by the xy-plane.

b. Find the volume of the upper region D cut from the solid sphere $\rho\le 1$ by the cone $\phi=\pi/3$, and, also find its mass and moment of inertia about the z-axis, density being $\delta=1$.

10. a. Integrate $f(x,y,z)=x-3y^2+z$ over the line segment C joining the origin and the point $(1, 1, 1)$.
b. A coil spring lies along the helix

$r(t)=(\cos 4t)i+(\sin 4t)j+t\,k,\ 0\le t\le 2\pi$.

The spring's density is constant $\delta=1$. Find the spring's mass and center of mass, and its moment of inertia and radius of gyration about the z-axis.

SECTION: C

(Long Answer Type Questions)

(Marks: 2X20=40)

11. a.(i) Find the circulation of the field $F(x,y)=(x-y)i+xj$ around the circle
$r(t)=(\cos t)i+(\sin t)j+t\,k,\ 0\le t\le 2\pi$.

(ii). Find the flux $F(x,y)=(x-y)i+xj$ of across the circle $x^2+y^2=1$ in the xy-plane.
b. State Green's Theorem for Flux-Divergence (or Normal Form) and Green's Theorem for Circulation –Curl (or Tangential Form).
Also verify both forms of Green's Theorem for the field $F(x,y)=(x-y)i+xj$ and the region bounded by the unit circle $C:\ r(t)=(\cos t)i+(\sin t)j,\ 0\le t\le 2\pi$.

12. a.(i) Find the area of the surface cut from the bottom of the paraboloid $x^2+y^2-z=0$ by the plane $z=4$.
(ii). Integrate over the surface $G(x,y,z)=xyz$ of the cube cut from the first octant by the planes $x=1,\ y=1$ and $z=1$.

b. Define flux in three dimension of a vector field. Find the flux of $F=yz\,j+z^2k$ outward through the surface S cut from the cylinder
$y^2+z^2=1,\ z\ge 0$ by the planes $x=0$ and $x=1$.

13. a. Verify Stokes theorem for the hemisphere $S:x^2+y^2+z^2=9,\ \ z\ge 0$ its bounding circle
$C:x^2+y^2=9,\ \ z=0$ and the field $F=yi-xj$.
b. Verify Gauss Divergence Theorem for the field $F=xi+yj+zk$ over the sphere $x^2+y^2+z^2=a^2$.