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DOON UNIVERSITY

M.Sc. Integrated (Mathematics)-II (Second Semester)
End-Semester Examination December 2017
Department of Mathematics, SOPS, Doon University Dehradun
Course: MAC-204, Mathematical Transforms

Time Allowed: 3 Hours | ***Maximum Marks: 100***

Note: *1. Attempt* ***All*** *Questions from* ***Sections A.***
2. Attempt ***any one*** *of* ***Q.1or Q.2 in section B.*** *Attempt* ***any four*** *questions from* ***Section C.***
3. ***Length*** *of each* ***answer*** *should be according to the* ***marks*** *allotted.*

Section A

Q.1. Answer in Brief: (2 marks each)

(i) Define convolution of two functions $f(t)$ and $g(t)$.

(ii) Write the z-transform of unit step function $u(t)$.

(iii) Write the domain of the function $\frac{1}{a+iw}, a > 0$, given $\mathcal{F}\{f(t)\} = F(w)$.

(iv) State final value theorem (formula only) of z-transforms.

(v) State first shift property (formula only) of Fourier transforms.

(vi) Define convergence of Fourier series at the point of discontinuity.

(vii) Write the formula for Fourier sine transform second derivative of a function.

(viii) What will be the value of $f(1)$ in terms of z-transform, when $f(0) = 0$.

(ix) Derive scaling property for Mellin transforms.

(x) Write main applications of Mellin and z-transform.

Section B

(8 marks each)

Q.1. (i) Find the Fourier series for $f(x) = \begin{cases} 0 \; ; & -\pi < x \leq 0 \\ \cos x \; ; & 0 \leq x < \pi \end{cases}$.

(ii) Find the Fourier cosine series for $f(x) = l - x$ on $0 \leq x \leq l$.

or

Q.2. (i) Find the Fourier series for $f(x) = x^2$ on $-l \leq x \leq l$.

(ii) Find the Fourier sine series for $f(x) = \begin{cases} \frac{\pi}{2}; & 0 \leq x \leq \frac{\pi}{2} \\ x - \frac{\pi}{2}; & \frac{\pi}{2} \leq x \leq \pi \end{cases}$.

Q.3. (i) Solve the differential equation $y' - 4y = H(t)e^{-4t}$ by applying Fourier transform.

(ii) Find the Fourier transform of $f(t) = \frac{1}{a+it}$ using symmetry property.

Q.4. (i) Prove that inverse Fourier transform of $1 = \delta(t)$, the dirac-delta function.

(ii) Find the inverse z-transform of $F(z) = \frac{z^2-1}{z^2+1}$.

Q.5. (i) Find the Mellin transform of $f(t) = (1 + t^a)^{-b}$ using the properties.

(ii) Use the property of derivatives to find the Mellin transform of $f(t) = a^2e^{-at}$.

Q.6. Find steady- state temperature distribution $u(x, y)$ in a thin, homogeneous, semi-infinite plate governed by

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0; 0 < x < l, 0 < y < \infty,\ u(0, y) = e^{-2y}, u(l, y) = 0, y > 0, \left(\frac{\partial u}{\partial y}\right)(x, 0) = 0.$$

Section C

(10 marks each)

Q.1. (i) Using Fourier expansion show that $1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} \ldots\ldots = \frac{\pi^2}{8}$.

(ii) Find the inverse Fourier transform of $F(w) = e^{-\left(\frac{w^2}{4}\right)}$.

Q.2. (i) Find the Fourier sine transform of $f(t) = e^{-2t}$ using formula for derivatives.

(ii) Find finite Fourier sine transform of $f(t) = \begin{cases} 0; & 0 \le x < \frac{\pi}{2} \\ 1; & \frac{\pi}{2} \le x < \pi \end{cases}$.

Q.3. (i) Prove that the Mellin transform of $f(t) = (e^x - 1)^{-1}$ is $\Gamma(s)\zeta(s)$, where $\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^2}$.

(ii) Find the Mellin transform of $f(t) = \cos at, a > 0$.

Q.4. (i) Show that $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^3} = \frac{3}{4}\zeta(z)$.

(ii) Prove that $M\left\{t^2 \frac{d^2 f(t)}{dt^2} + t\ \frac{df(t)}{dt}\right\} = z^2F(z)$.

Q.5. (i) Solve the difference equation $f(n + 2) - 5f(n + 1) + +6f(n) = 2^n$ using z-transforms with $u(0) = 1, u(1) = 0$.

(ii) Using initial value theorem find $f(0)$, where $F(z) = \frac{z}{(z-1)^2}$.