11-12-17

I.M.Sc. (Mathematics)-II (Third Semester)
End-Semester Theory Examination 2017-18
Department of Mathematics, SOPS, Doon University Dehradun
Core Course, Partial Differential Equations & System of ODE (MAC-203)

Time: 03 Hour **Total Marks: 60**

Note: (i) Attempt ALL the questions. (ii) Do neat and clean work.

Section A

Attempt ALL: **(1x10=10)**

1. Subsidiary equations for the equation $\left(\frac{y^2z}{x}\right) + zxy = y^2$ *are* −
   (a) $\frac{dx}{y^2z} = \frac{dy}{zx} = \frac{dz}{y^2}$; (b) $\frac{dx}{x^2} = \frac{dy}{y^2} = \frac{dz}{zx}$; (c) $\frac{dx}{x^2} = \frac{dy}{y^2} = \frac{dz}{zx}$; (d) $\frac{dx}{1/x^2} = \frac{dy}{1/y^2} = \frac{dz}{1/zx}$
2. The equation $Pp + Qq = R$ is known as-
   (a) Charpits equation; (b) Lagrange' equation; (c)Bernoulli' equation; (d) Clairaut's equation
3. Solution for the partial differential equation $x^2p + yq = (x - y)z^2 + x - y$ *is*-
   (a)Quasi-linear; (b) Semi-linear; (c) Linear; (d) non-linear
4. The PDE of the form $f(x, y, z)\frac{\partial z}{\partial x} + g(x, y, z)\frac{\partial z}{\partial y} = h(x, y, z)$ is called _____
5. How do you classify a PDE of second order with variable coefficients?
6. In the PDE $f(D, D')z = f(x, y)$. DD'*means*_____
7. PI of the equation $(r - 2s + t) = cos(2x + 3y)$ is_____
8. When a vibrating string has an initial velocity, its initial conditions are_____
9. Two dimensional steady state heat flow is given by Laplace equation $u_t = a^2(u_{xx} + u_{tt})$ (T/F)
10. The PDE $\frac{\partial y}{\partial x} = c^2\frac{\partial^2 u}{\partial x^2}$ is of _____type.

Section B

Attempt any FIVE: **(5x4=20)**

1. Find the integral surface of $px^2 + qy^2 + z^2 = 0$, which passes through the hyperbola $xy = x + y, z = 1$.
2. (a) Solve the PDE $p^3{}_1 + p^3{}_2 + p^3{}_3 = 1$
   (b) Solve the PDE $16p^2z^2 + 9q^2z^2 + 4z^2 - 4 = 0$
3. Discuss the singular integral of a partial differential equation by inspection method.
4. Find the complete Integral of the PDE $xp^3q^2 + yp^2q^3 + (p^3 + q^3) - zp^2q^2 = 0$.
5. (a) Define the following with the help of example:
   (i) Complete Integral, (ii) General Integral, (iii) Singular Integral
   (b) Find the complete integral of the PDE $(x^2 - y^2)pq - xy(p^2 - q^2) = 1$
6. Show that the following system of partial differential equation are compatible and hence solve them $p = x^4 - 2xy^2 + y^4; q = 4xy^3 - 2x^2y - siny$.

Section C

Attempt ALL: **(5x6=30)**

1. A homogenous rod of conducting material of length l has its ends kept at zero temperature. The temperature at the center is T and falls uniformly to $z = 0$ at the two ends. Find the temperature function $u(x, y)$.

OR

A string of length l is initially at rest in its equilibrium position and motion is started by giving each of its points a velocity v is given by $v = k(l - x)\ if\ \frac{l}{2} \leq x \leq l$. Find the displacement function $y(x, t)$.

2. (a) A surface is drawn satisfying $r + t = 0$ and touching $x^2 + y^2 = 1$ along its section by $y = 0$. Obtain its equation in the form $x(x^2 + z^2 - 1) = y^2(x^2 + z^2)$

   (b) find the solution of the PDE $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} + \frac{\partial z}{\partial x} + 3\frac{\partial z}{\partial y} - 2z = e^{x-y}$.

3. (a) Solve $(D^3 - 7DD'^2 - 6D'^3)z = x^2 + xy^2 + y^3 + \cos(x - y)$

   (b) Find the solution of the equation $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = e^{-x} cosy\ which\ \to 0\ as\ x \to \infty$ and has the value $cosy\ when\ x = 0$.

4. (a)Write down the system of equations for obtaining the general equation of surfaces orthogonal to the family given by $x(x^2 + y^2 + z^2) = c_1 y^2$.

   (b) Solve the PDE $(x + y - z)(p - q) + a(px - qy + x - y) = 0$

5. Reduce the equation $y^2 \frac{\partial^2 z}{\partial x^2} - 2xy \frac{\partial^2 z}{\partial x \partial y} + x^2 \frac{\partial^2 z}{\partial y^2} = \frac{y^2}{x}\frac{\partial z}{\partial x} + \frac{x^2}{y}\frac{\partial z}{\partial y}$ into canonical form and hence solve it.

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