8-12-2017

DOON UNIVERSITY
End Semester Examination, 2017
B. Sc.(I)
Course: MAC-101: Calculus

Time Allowed: 3 Hours **Maximum Marks: 100**

Note: *Attempt all questions from section A, any five questions from section B and any two questions from section C.*

Section: A (5×4 = 20 Marks)

1. Expand the polynomial $-35 + 21x + 12x^2 - 8x^3 + x^4$ in powers of $(x - 2)$.
2. Show that the arc of the upper half of the cardioid $r = a(1 - \cos\theta)$ is bisected by $\theta = 2\pi/3$.
3. Find the surface area of the solid generated by the revolution of the curve $x^{2/3} + y^{2/3} = a^{2/3}$ about the $x-$axis.
4. If the pressure function on Nilrebo is $f(x, y, z) = 5x^2 + 7y^4 + x^2z^2$ atm, where the origin is located at the center of Nilrebo and distance units are measured in thousands of kilometers, then find the rate of change of pressure (i.e. directional derivative)at $(1, -1, 2)$ in the direction of $\hat{i} + \hat{j} + \hat{k}$.

Section: B (8×5 = 40 Marks)

1. If $P_{n+1}(x) = \dfrac{1}{2^n\, n!}\dfrac{d^n}{dx^n}[(x^2 - 1)^n]$, prove that
$$P'_{n+1}(x) = xP_n(x) + (n + 1)P_n(x).$$
2. Find all the asymptotes of the curve
$$(x^2 - y^2)(x + 2y) = y^2 - y + 1.$$
3. If $u_n = \displaystyle\int_0^{\pi/2} x^n \sin x\, dx, n > 1$, prove that
$$u_n + n(n - 1)u_{n-2} = n\left(\frac{\pi}{2}\right)^{n-1}.$$
Hence evaluate $\displaystyle\int_0^{\pi/2} x^5 \sin x\, dx$.
4. Trace the curve $8a^2y^2 = x^2(a^2 - 2x^2)$ and show that the whole length of the curve is $a\pi$.
5. Show that the larger of the two areas into which the circle $x^2 + y^2 = 64a^2$ is divided by the parabola $y^2 = 12ax$ is $\frac{16}{3}a^2(8\pi - \sqrt{3})$.

6. By an induction on n, prove $\nabla(f^n) = nf^{n-1}\nabla f$.

Section: C

(20 × 2 = 40 Marks)

1. (a) Compute $y_n(0)$ for $y(x) = \tan^{-1} x$ and use the Maclaurin's expansion prove that
$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \cdots + (-1)^{n-1}\frac{1}{2n-1} + \cdots$$
   (b) Show that the necessary and sufficient condition that the vector $\vec{a}(t)$ be of constant magnitude is
$$\vec{a} \cdot \frac{d\vec{a}}{dt} = 0.$$

2. If $r = f(\theta)$ be a curve, then show that the sectorial area bounded by $\theta = \alpha$, $\theta = \beta$ and the curve is given by
$$\frac{1}{2}\int_{\alpha}^{\beta} r^2\, d\theta.$$
Hence, find the area of one loop of the curve $r = a\sin 3\theta$.

3. Find $\int_C x^2 y\, dx - (x+y)dy$, where C is the trapezoid with vertices $(0,0)$, $(3,0)$, $(3,1)$, and $(1,1)$, oriented counterclockwise.