7-1217

DOON UNIVERSITY, DEHRADUN
End Semester Examination, Third Semester, 2017
School of Physical Sciences
Integrated M.Sc.(Mathematics)
Course: MAC-202: Group Theory I

Time Allowed: 2 Hours | **Maximum Marks: 50**

Note:

1. **Attempt any nine Questions from Sections A.**
2. **Attempt any four Questions from Sections B.**
3. **Attempt any two Questions from Sections C.**

SECTION: A

($9 \times 2 = 18$ Marks)

1. The order of the permutation (124)(3567) is....
2. Let a be any element of order n in a group G and let k be a positive integer. Then $< a^{gcd(n,k)} >$ $= \ldots$ and $o(a^k) = \ldots.$
3. A group of order 12 may not have a subgroup of order....
4. Any group of order 15 is ....
5. If the order of $\frac{G}{Z}$ is 77, then G is ....
6. In a finite group, the number of elements of order d is a multiple of ....
7. For every integer a and every prime p, $a^{p-1} \equiv \ldots$
8. Let H be a subgroup of a group G and $a, b \in G$. Then $aH = bH$ iff ....
9. Let $G = S_3$ and $N = \{I, (123), (132)\}$. Then $\frac{G}{N}$ is ... group.
10. Any infinite cyclic group is isomorphic to ....

SECTION: B

($5 \times 4 = 20$ Marks)

1. If Z is the centre of a group G such that $\frac{G}{Z}$ is cyclic. Then show that G is abelian.
2. Show that every homomorphic image of a cyclic group is cyclic. Is converse of this result true? Give an example.
3. Show that every subgroup of an abelian group is abelian.Give an example to show that the converse need not be true.
4. Show that a subgroup H of a group G is normal iff $Ha \neq Hb \Rightarrow aH \neq bH$.
5. Prove that a group G is abelian iff the mapping $f : G \to G$ such that $f(x) = x^2$ is a homomorphism.

SECTION: C

($2 \times 6 = 12$ Marks)

1. Prove that H is a normal subgroup of a group G iff the product of any two right cosets of H in G is again a right cosets of H in G.
2. State and prove of the fundamental theorem of homomorphism.
3. If a is any element of a group G. Show that $o(a^n) = \frac{o(a)}{(n, o(a))}$.