23/3/17

DOON UNIVERSITY, DEHRADUN

Mid Semester Examination, second Semester, 2017

School of Physical Sciences

M.Sc.(Mathematics)

Course: MAC-454: Measure & Integration

Time Allowed: 2 Hours Maximum Marks: 60

Note:

1. Attempt all Questions from Sections A.
2. Attempt any five Questions from Sections B.
3. Attempt any three Questions from Sections C.

SECTION: A

($6 \times 1 = 6$ Marks)

1. If A is a countable, then outer measure of A i.e. $m^*(A) =$ .....
2. If C denote the Cantor set, then the exterior measure of C is ....
3. If α_0 being finite cardinal number, then $\alpha_0 + 1 =$....
4. A set E is measurable iff for any set A....
5. If A_i is countable infinite set, then $\cup_{i \in \mathbb{N}} A_i$ is .... and n α_0=....
6. A property is said to hold almost everywhere (abbreviated a.e.) if the set of points where it fails to hold is a set of.....

SECTION: B

($5 \times 6 = 30$ Marks)

1. Define interior and exterior measure of a set and prove that $m_e(A) \geq m_i(A)$.
2. Show that a continuous function defined over a measurable set is measurable.
3. If f and g are measurable functions defined on a measurable set E,then show that $|f|$ and $\frac{f}{g}$ (g vanish nowhere on E) are measurable over E.
4. Prove that a.a = a, where a being cardinal number of $\mathbb{N}$
5. A function f is measurable iff the set $\{x : f(x) < r\}$ is measurable for every rational number r.
6. Show that the set of all rational numbers is countable.

SECTION: C

(3 × 8 = 24 Marks)

1. If E_1 and E_2 are measurable subsets of [a,b], then prove that $m(E_1) + m(E_2) = m(E_1 \cup E_2) + m(E_1 \cap E_2)$.

2. Suppose that $< E_n >$ is a monotonic increasing sequence and E is their limiting sum then show that $m(E) = Lim_{n \to} m(E)$.

3. If $E_1, E_2, ...$ are disjoint measurable sets and $E = E_1 + E_2 + ...$Show that E is measurable and $m(E) = \sum_{k=1}^{\infty} m(E_k)$.

4. Construct Cantor set and prove that it is measurable and find its measure.