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DOON UNIVERSITY, DEHRADUN

Mid Semester Examination, Fourth Semester, 2017

School of Physical Sciences

Integrated M.Sc. (Mathematics)

Course: MAC-253: Ring Theory & Linear Algebra

Time Allowed: 2 Hours | Maximum Marks: 60

Note:

1. **Attempt any fifteen Questions from Sections A.**
2. **Attempt any five Questions from Sections B.**

SECTION: A

($15 \times 2 = 30$ Marks)

1. If p is a prime, then the number of divisors of zero in Z_p are ....

2. The number of non-zero nilpotent elements in Z_6, the ring of integers modulo 6 are .....

3. Let Z be the ring of integers and $<p>$ be an ideal of Z consisting of all multiples of a prime p. Let $<n>$ be an ideal of Z such that $<p>\subset<n>\subset Z$, Then .... or ....and $<p>$ is .... ideal of Z.

4. If $f: Z_2 \to Z_2$ defined by $f(n) = n^2 - n$ is a ring homomorphism. Then kernel of f is ....

5. Let E be the ring of even integers and $<4>$ is an ideal of E. Then I is a ... ideal of E.

6. If f is a homomorphism from a ring R onto a field F. Then Kernel of f is ... ideal of R.

7. If R be a commutative ring with unity, then every .... ideal of R is a ... ideal of R. item In Z_6, the ring of integers modulo 6, the number of divisors of zero are ....

8. In Z_5, the ring of integers modulo 5, the only idempotent elements are ... and the only nilpotent elements are ...

9. Let $V = R^3$ be vector space, $W \subseteq V$ such that $W = \{(a_1, a_2, a_3) : a_2a_3 = 0; a_1, a_2, a_3 \in R\}$ is ... of R^3.

10. The sets $W_1 = \left\{ \begin{pmatrix} 0 & a \\ 0 & a \end{pmatrix} |a \in \mathbb{R} \right\}$ and $W_2 = \left\{ \begin{pmatrix} a & 0 \\ a & 0 \end{pmatrix} |a \in \mathbb{R} \right\}$ are subspaces of the vector space M_{22} of 2×2 matrices over R.Then $W_1 \cup W_2$ is ... of $M_{2,2}$.

11. The sum of two nilpotent elements of a ring R is .... nilpotent.(necessarily/not necessarily)

12. Let V be a vector of all $n \times n$ matrices over the field R.Let W_1 be the set of all $n \times n$ symmetric matrices in V and W_2 be the set of all skew-symmetric matirces in V.Then V can be represented in terms of W_1 and W_2 as ....

13. Let A be the ring of all continuous real valued functions, defined on [0,1] and R the field of real numbers. The mapping $\phi : A \to R$ defined by $\phi(f(x)) = f(\frac{1}{2})$ for all $f(x) \in A$ is onto homomorphism. Then the kernel of ϕ is ....

14. If $\mathbb{Z}$ being a commutative ring with unity,then the zero ideal $< 0 >$ is ... ideal but not ... ideal of $\mathbb{Z}$.

15. Is the sum $\mathbb{R}^3 = xy - plane + yz - plane$ direct?....If no write it more than one way.

16. If $C[a, b]$ be the vector space,Is the subset $\{f \in C[a, b] | f(\frac{a+b}{2} = 1)\}$ form a subspace of $C[a, b]$?...

SECTION: B

(5 × 6 = 30 Marks)

1. State and prove of the fundamental theorem of homomorphism.

2. Show that the homomorphic image of a commutative ring is a commutative ring,but converse need not be true.

3. (i)Prove that a finite integral domain is a field.
   (ii) Show that a ring R iff $a^2 - b^2 = (a + b)(a - b)\ \forall\ a, b \in R$.

4. (i) Prove that any homomorphism of a field is either an isomorphism or takes each element into 0.
   (ii)What can you say about the sum of two subrings of a ring?

5. If U is an ideal of R, then prove that $r(U) = \{x \in R : xu = 0 \forall u \in U\}$ is an ideal of R.

6. Let R be a commutative ring. Prove that an ideal P of ring R is a prime ideal iff R/P is an integral domain.