20-3-17

DOON UNIVERSITY, DEHRADUN

Mid Semester Examination, Second Semester, 2016-17

School of Physical Sciences

Class: Integrated M.Sc. Mathematics **Course: Real Analysis**

Semester: II **Course Code: MAC-151**

Time Allowed: 2Hours *Maximum Marks: 30*

Note: **Attempt all six questions in Section A. Each question carries 1 marks.**
Attempt any four questions in Section B. Each question carries 3marks.
Attempt any two questions in Section C. Each question carries 6 marks.

SECTION: A
(Very Short Answer Type Questions)

(Marks:6X1=6)

1. Define the following terms:
   (a) Compact set (b) Bounded sequence
2. Prove that if x is a limit point of A and $A \subset B$, then x is also a limit point of B.
3. The set of irrational numbers is uncountable.
4. Find the $l.u.b.$ and $g.l.b.$ if they exist, of the following two sets:
   (a) $A = \{(-1)^n n : n \in N\}$ (b) $B = \{1 + \frac{1}{n} : n \in N\}$
5. The union of two closed sets is a closed set.
6. If the sequence $< a_n >$ converges to l, then the sequence $< a_n >$ converges to $|l|$.

SECTION: B
(Short Answer Type Questions)

(Marks: 4X3=12)

7. Prove that the set of real numbers x such that $0 \leq x \leq 1$ is not countable.
8. A is open *iff* $A = A^o$, where A is any subset of R.
9. Prove that between two different rational numbers, there lie an infinite number of rational numbers.
10. Show that $\lim_{n\to\infty} \frac{1+2+3+\cdots..n}{n^2} = \frac{1}{2}$.
11. Prove that a sequence $< a_n >$ is bounded if and only if there exists a positive real number M such that $|a_n| \leq M \; \forall \, n \in N$.

SECTION: C
(Long Answer Type Questions)

(Marks: 2X6=12)

12. (a) $(A \cup B)' = A' \cup B'$
    (b) Prove that $\sqrt{3}$ is not a rational number.
13. (a) Show that $|a + b| = |a| + |b|$ *iff* $ab < 0$.
    (b) Prove that every convergent sequence has a unique limit.
14. (a) Prove that a set A is closed if and only if $A = \bar{A}$.
    (b) Every monotonically increasing sequence which is bounded above converge to its least upper bound.