18-12-2015

Integ. MCA.-I SEMESTER EXAMINATION 2015-16

Department of Mathematics, SOPS, Doon University Dehradun

Mathematics-I (STM-502)

Time: 03 Hour **Total Marks: 100**

Note: (i) Attempt ALL the questions. (ii) Do neat and clean work.

Roll No: | | | | | | | | | | |

SECTION A

Attempt any FOUR: **(5x4=20)**

1. Find the nth derivative of $tan^{-1}\left(\frac{2x}{1-x^2}\right)$.
2. If $y = \sin(msin^{-1}x)\ then,$
   $prove\ that\ (1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2 - m^2)y_n = 0.$
3. Find the asymptotes of the curve$x^3 + 2x^2y - xy^2 - 2y^3 + 4y^2 + 2xy + y - 1 = 0.$
4. Prove that $A - A'$ is skew –symmetric, if A is a square matrix.
5. By the elementary transformations , find the inverse of the
   matrix $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

SECTION B

Attempt any FOUR: **(5x4 =20)**

1. Find the rank of the matrix $A = \begin{bmatrix} 1^2 & 2^2 & 3^2 \\ 2^2 & 3^2 & 4^2 \\ 3^2 & 4^2 & 5^2 \end{bmatrix}$
2. Show that$\int_0^1 dx \int_0^1 \frac{x-y}{(x+y)^3} dy \neq \int_0^1 dy \int_0^1 \frac{x-y}{(x+y)^3} dx.$
3. Prove that $\beta(m,n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}} dx$
4. Prove that $\int_0^{\frac{\pi}{2}} \sqrt{tan\theta} d\theta = \int_0^{\frac{\pi}{2}} \sqrt{cot\theta} d\theta = \frac{\pi}{\sqrt{2}}$
5. Prove that $\frac{x}{1+xtanx}$ is a maximum when $x = cosx$

SECTION C

Attempt any FOUR: **(5x4 =20)**

1. Find the eigenvalue and eigenvector of the
   matrix:$\begin{bmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{bmatrix}$
2. Find the n[th] derivative of the function $y = \frac{x}{x^2+a^2}$
3. $Prove\ that\ \beta(m,n) = \frac{\Gamma m.\Gamma n}{\Gamma(m+n)}$
4. If $u = x + y + z, v = x^2+y^2+z^2, w = xy + yz + zx,$
   prove that $gard\ u, gradv, gradw$ are coplanar vectors.

5. Find values of λ for which the following system of equations has a non-trival solution $3x + y - \lambda z = 0,$

$$2x + 4y + \lambda z = 0,$$

$$8x - 4y - 6z = 0$$

(*True or False*)

6. Define the condition for the consistency of the linear homogeneous simultaneous equation.
7. Expand $e^x cosx$ in powers of x and y as far as the terms of third degree.
8. Expand $sinxy$ *in powers of* $(x-1)$*and* $(y-\frac{\pi}{2})$upto second degree terms.

SECTION D

Attempt any TWO: **(10x2=20)**

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1. State and prove Leibnitz theorem.
2. Evaluate $\iint (x+y)^2 dxdy$ over the area bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
3. Prove that

$\Gamma(m)\Gamma(m+1) = \frac{\sqrt{\pi}}{2^{2m-1}}\Gamma(2m)$*where* m *is positive.*

SECTION E

Attempt ALL **(from 1-6:1mark & 7-8:2mark)**

1. What do you understand by irrotational motion?
2. Define the divergence of a vector point function.
3. The gradient of $2x^2z\hat{\imath} - xy^2z\hat{\jmath} + 3yz^2\hat{k}$ *at the point* $(1,1,1)$*is*_____
4. The value of $\Gamma\frac{1}{3}.\Gamma\frac{2}{3} =$ _____
5. If$|A| = 0$, *then at least one eingen value is zero.*