21-12-2015

Roll No.: ....

DOON UNIVERSITY, DEHRADUN

End Semester Examination, First Semester, 2015
School of Physical Sciences

M. Sc. - Physics
Course: PHC-404: Quantum Mechanics

Time Allowed : 3 *Hours* *Maximum Marks* : 50

Note: Attempt All Sections - A, B, C.

SECTION: A (Very Short Type Questions). All questions are compulsory.

(Marks: $1.5 \times 8 = 12$)

1. What is the difference between classical mechanics and quantum mechanics?

2. Write the physical significance of the integral $\int_{-\infty}^{+\infty} \psi^* \psi \, d\tau = 0$.

3. Write the validity of Schrodinger's wave equation.

4. What bearing would you think the uncertainty principle has on the existence of the zero-point energy of a harmonic oscillator?

5. Write a Hamiltonion for a freely moving particle.

6. What is Gaussian wave packet?

7. Define the projection operator. Explain that the sum of two projection operators is generally projection or not.

8. Write the normalized wave function of the one dimensional harmonic oscillator.

SECTION: B (Short Answer Type Questions). Attempt any *five* questions.

(Marks: $4 \times 5 = 20$)

9. Explain all the quantum effect which shows the particle nature of radiation.

10. If $\hat{X}$ and $\hat{Y}$ are two operators such that $[\hat{X}, \hat{Y}] = 1$, find out the value of $[\hat{X}, \hat{Y}^2]$.

11. Explain the bound and unbound states of a single particle moving in one dimensional potential.

12. A particle of mass m is in the state $\psi(x,t) = A \exp -a[(mx^2/\hbar) + it]$, where A and a are the positive constants. Find A.

13. Find out the n^{th} wave function of the infinite square well time dependent potential ($V(t)$).

14. A particle state is in the linear combination of just two stationary states:

$$\psi(x,0) = c_1\psi_1(x) + c_2\psi_2(x),$$

what is the wave function $\psi(x,t)$ at subsequent times? Find the probability density and describe its motion.

15. Verify that the average value of $1/r$ for a $1s$ electron in the hydrogen atom is $1/a_0$.

SECTION: C (Long Answer Type Questions). Attempt any *three* questions.

(Marks: $6 \times 3 = 18$)

16. **(a)** A particle limited to the x-axis has the wave function $\psi = ax$ between $x = 0$ and $x = 1$; $\psi = 0$ elsewhere. Find the probability that the particle can be found between $x = 0.45$ and $x = 0.55$, and also find the expectation value $< x >$ of the particle's position.

    (b) Find the expectation value $< x >$ for the first two states of a harmonic oscillator.

17. A particle is moving in the potential well

$$V(x) = \begin{cases} 0 & x < 0, \\ V_0 & 0 \leq x \leq a, \\ 0 & x > a. \end{cases}$$

where V_0 is positive. Sketch and obtain an exact solution of the Schrodinger equation and an expression for the transmission coefficient.

18. A particle of mass m, which moves freely inside an infinite potential well of length a, is initially in the state

$$\psi(x,0) = \sqrt{\frac{3}{5a}} sin(3\pi x/a) + \sqrt{\frac{1}{5a}} sin(5\pi x/a).$$

(a) Find $\psi(x,t)$ at any later time t.
(b) Calculate the probability density $\rho(x,t)$ and the current density $\vec{J}(x,t)$.

19. A azimuthal wave function for the hydrogen atom is $\Phi(\phi) = Ae^{im_l\phi}$. Find the value of normalization constant. If the wave function of the hydrogen atom for $1s$ state is

$$\Psi(1s) = \frac{1}{\sqrt{\pi}} \left(\frac{1}{a_0}\right)^{3/2} e^{-r/a_0}$$

where $a_0 = \hbar^2/me^2$ is the Bohr's radius. Calculate the expectation value of the potential energy of electron in $1s$ state (only algebraic expression is required, not the numerical answer).

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