18-12-2015

DOON UNIVERSITY, DEHRADUN

End Semester Examination, First Semester, 2015

Department of Physics, School of Physical Sciences

M.Sc. Physics (Integrated) 5 Years

Course: PHC-101: Mathematical Physics - I

Time Allowed: 3 Hours *Maximum Marks: 30*

Note: Attempt All Questions from Sections A,B,C.

SECTION: A

Attempt All Questions. *(Marks:1 X 6 =6)*

1. Find out the differential of the function $f(x,y) = y\exp(x + \tan^{-1} y)$
2. What are the point of discontinuity of the function 1/{exp(sin(log x))-1}
3. Find out the first and second order partial derivatives of the function $f(x,y) = 2x^3 y + y^3$
4. Show whether the below differential is exact or inexact:
   (y + z)dx + xdy + 3xdz
5. Differentiate the vector $\vec{r}(t) = 2t^2\hat{i} + (3t-2)\hat{j} + (3t^2-1)\hat{k}$ with respect to t. Find out the angle between vector and its derivative at t = 1.
6. Find out the curl of the vector field of the vector $\vec{a} = x^2y^2z^2\hat{i} + y^2z^2\hat{j} + x^2z^2\hat{k}$

SECTION: B Attempt All Questions. *(Marks: 3 X 4=12)*

7. Express the spherical polar coordinates in terms of Cartesian coordinates. Also, express the unit vectors of spherical polar coordinates in terms of $\hat{i}$, $\hat{j}$ and $\hat{k}$. Hence, express the volume element in spherical polar coordinates.
8. Find out the solution and the Wronskian of the following second order differential equation: $\frac{d^2y}{dx^2} + 3y = 0$
9. Prove the Green's theorem in two dimensions.
10. Prove that the two sets of basis vectors $\{\vec{e}_i\}$ and $\{\vec{\varepsilon}_i\}$ are reciprocal sets in curvilinear coordinate system corresponding to contravariant and covariant components, respectively.

SECTION: C Attempt All Questions. *(Marks: 4X 3=12)*

11. Establish the unit vector formalism of curvilinear coordinates. Hence, express the vector displacement dr, length element ds^2 and volume element dV. Also, write the expression for gradient of a quantity φ in terms of curvilinear coordinate system vectors.

12. Find the stationary points of the function $f(x,y,z) = x^3 + y^3 + z^3$, subject to the constraint $g(x,y,z) = x^2 + y^2 + z^2 = 1$

13. Evaluate the line integral $I = \int_C \vec{a} \bullet d\vec{r}$, where $a = (x+y)\hat{i} + (y-x)\hat{j}$ along each of the paths in the xy-plane:

~~(a) The parabola $y^2 = x$ from (1,1) to (4,2).~~

(b) The curve $x = 2u^2 + u + 1$ and $y = 1 + u^2$ from point (1,1) to (4,2)

(c) The line $y = 1$ from (1,1) to (4,1) and then line $x = 4$ from (4,1) to (4,2)