16/12/2015

DOON UNIVERSITY, DEHRADUN

End Semester Examination, First Semester, 2015

School of Physical Sciences

M.Sc.

Course: MAG-102: Finite Element Method

Time Allowed: 3 Hours **Maximum Marks: 100**

Note:
Attempt all questions from Sections A.
Attempt any four questions from Sections B.
Attempt any two questions from Sections C.

Section: A ($5 \times 4 = 20$ Marks)

(1) Write the Forward approximation formula for $\dfrac{du}{dx}$ and $\dfrac{d^2u}{dx^2}$at $x = x_i$.

(2) Write down the Crank-Nicholson formula to solve $\dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial u^2}$.

(3) Consider the differential equation defined on $[0,1]$ with its boundary conditions (which represents a diffusion-reaction equation in chemistry) given by

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - y = 0, y'(0) = 0, y(1) = 1.$$

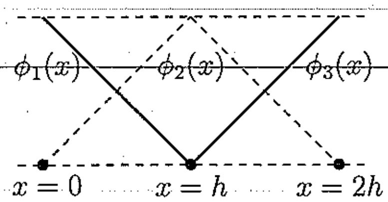
By choosing $\phi_1(x) = 1$ and $\phi_2(x) = (1 - x^2)$ as the basis function, find the weighted residual $R(x)$.

(4) For the two-point BVP

$$u''(x) = f(x), 0 < x < 1, u(0) = 0, u(1) = 0,$$

construct a variational or weak formulation.

(5) Write the formula for the shape functions, $\phi_1(x)$ and $\phi_2(x)$ shown in following figure



Section: B ($10 \times 4 = 40$ Marks)

(1) Apply collocation method to solve the BVP

$$\frac{d^2y}{dx^2} - y = 0, y(0) = 0, y(1) = 1$$

by taking $\phi_1(x) = x(1-x)$, $\phi_2(x) = x^2(1-x)$ and $\phi_3(x) = x^3(1-x)$.

(2) Solve the BVP of (1) by Galerkin method and taking only $\phi_1(x)$ and $\phi_2(x)$ as the basis functions.

(3) By Cranck-Nicholson method solve the equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2},$$

subject to $u(x,0) = 0, u(0,t) = 0$ and $u(1,t) = t$ for two time steps.

(4) Suppose the weighted residual for a BVP is given by

$$\int w(x)\left(\frac{d^2y}{dx^2} - x - 1\right) dx = 0,$$

where $y = x + C_1(x^2 - x) + C_2(x^3 - x)$. Taking $w(x) = x^2 - x$, and $x^3 - x$ find C_1 and C_2.

Section: C (20 × 2 = 40 Marks)

(1) Solve

$$\frac{\partial^2 u}{\partial x^2} = 2\frac{\partial u}{\partial t},$$

given $u(0,t) = 0, u(4,t) = 0, u(x,0) = x(4-x)$ taking $\Delta x = \Delta t = 1$. Find the value of u upto $t = 2$ using Bender-Schmidt explicit difference scheme.

(2) Using the five point formula

$$u_{i,j} = \frac{1}{4}\left(u_{i+1,j} + u_{i-1,j} + u_{i,j+1} + u_{i,j-1}\right)$$

solve the elliptic PDE

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0; 0 < x < 1; 0 < y < 1.$$

Given $u(x,0) = 10x, u(0,y) = 10y, u(1,y) = 10, u(x,1) = 10$ by taking $\Delta x = h = 1/3 = \Delta y = k$.

(3) Solve the BVP

$$y'' - 2y = 4; y(0) = 1, y(1) = 4,$$

by FEM upto the step giving two equations in C_1 and C_2 without evaluating the integrals.