21-12-2015

DOON UNIVERSITY, DEHRADUN

Final Semester Examination, First Semester, 2015

School of Physical Sciences

Class: M.Sc. Mathematics | **Course: Numerical Analysis**

Semester: I | **Course Code: MAC-404**

Time Allowed: 3Hours | *Maximum Marks: 60*

Note: Attempt all six questions in Section A. Each question carries 2 marks.
Attempt any four questions in Section B. Each question carries 6 marks.
Attempt any three questions in Section C. Each question carries 8 marks.

SECTION: A

(Very Short Answer Type Questions)

(Marks:6X2=12)

1. Using Newton's divided difference interpolation, find the polynomial of the given data:

x	-1	0	1	3
$f(x)$	2	1	0	-1

2. If $y' = x + 2y,\ y(0) = 2$ then by Picard's method, the value of $y^{(2)}(x)$ is.........
3. Perform only three iterations of Regula-falsi method to find the real root of the equation $x^3 - 2x^2 - 5 = 0$ in the interval $[1,4]$.
4. Derive linear Lagrange's interpolation formula.
5. Define the terms zero stability and root condition for a linear multi-step method for the initial value problem $y' = f(x,y),\ y(x_0) = y_0$.
6. Define linear and nonlinear boundary value problems of second order with Dirichlet and non-Dirichlet boundary conditions.

SECTION: B

(Short Answer Type Questions)

(Marks: 4X6=24)

1. Obtain the piecewise cubic interpolating polynomial for the function $f(x)$ defined by the given data.

x	-5	-4	-2	0	1	3	4
$f(x)$	275	-94	-334	-350	-349	-269	-94

2. Obtain quadratic spline approximation for the function given by following data

x	-1	0	1	2
$f(x)$	-4	1	0	5

assume $f''(2) = M(2) = 0$. Hence find an estimate of $f(-0.5)$.

3. Given $y' = x + \sin y,\ y(0) = 1$. Compute $y(0.2)$ and $y(0.4)$ with $h = 0.2$ using Euler's modified method.

4. The boundary-value problem

$$y'' = 4(y - x),\ \ 0 \le x \le 1,\ \ y(0) = 0,\ \ y(1) = 2.$$

has the solution $y(x) = e^2(e^4 - 1)^{-1}(e^{2x} - e^{-2x}) + x$. Use the Linear Finite-Difference method to approximate the solution, and compare the results to the actual solution. Take $h = 0.25$.

5. Show that the order of the linear multi-step ,method

$$y_{j+1} + (a-1)y_j - ay_{j-1} = \frac{h}{4}[(a+3)y'_{j+1} + (3a+1)y'_{j-1}]$$

is 2 if $a \neq -1$ and it is 3 if $a = -1$. Find the value of a for which the root condition holds.

SECTION: C
(Long Answer Type Questions)

(Marks: 3X8=24)

1. Derive formula for Hermite interpolating polynomial $p(x)$ for the function $f(x)$ with interpolating conditions $p(x_i) = f(x_i)$ and $p'(x_i) = f'(x_i)$, $i = 0,1,\dots n$. Hence,find the value of $f(1.05)$ from the following data:

x	0	1	2
$f(x)$	0	1	0
$f'(x)$	0	0	0

2. Using Runge-Kutta method of order four, calculate $y(0.1), y(0.2), y(0.3), y(0.4)$ given that $y' = x^2 - y^2$, $y(0) = 1$. Taking these values as starting values, find $y(0.5)$ correct to three decimal places using Adams–Bashforth and Adams–Moulton methods as predictor-corrector methods respectively.

3. Use Newton's method with $x^{(0)} = 0$ to compute $x^{(3)}$ for the following nonlinear system.

$$4x_1^2 - 20x_1 + \frac{1}{4}x_2^2 + 8 = 0$$

$$\frac{1}{2}x_1x_2^2 + 2x_1 - 5x_2 + 8 = 0.$$

4. Find the interval of absolute stability for the two-step Adams–Bashforth method $y_{n+2} - y_{n+1} = \frac{h}{2}[3f_{n+1} - f_n]$ using Schur's criterion and the Routh–Hurwitz criterion.