M.Sc.(Mathematics)-I

SEMESTER EXAMINATION 2015-16

Department of Mathematics, SOPS, Doon University Dehradun

Core Course-III, ODE (MAC-403)

Time: 03 Hour   Total Marks: 60

Note: (i) Attempt ALL the questions. (ii) Do neat and clean work.

Section A

Attempt any ALL: (2x5=10)

1. Define Spiral point with example.

2. Modeling the free motion of a mass on a spring and draw the trajectories.

3. Find the eigenvalues and eigenvectors & draw the trajectories for the system $y' = Ay = \begin{bmatrix} -3 & 1 \\ 1 & -3 \end{bmatrix} y$

4. Show that linearly independent solutions of $y'' - 2y' + 2y = 0$ are $e^x \sin x$ and $e^x \cos x$. What is the general solution? Find the solution $y(x)$ with the property $y(0) = 2, y'(0) = 3$.

5. Define proper Node.

Section B

Attempt any THREE: (10+5x2=20)

*Ques.1 is compulsory.

1. State and prove Lipschitz theorem.

2. Find the third approximation of the solution of the equation $\frac{dy}{dx} = 2x + z, \frac{dz}{dx} = 3xy + x^2 z$ where $y = 2$ and $z = 0$ when $x = 0$

3. Test the exactness and solve $(1 + x^2)y'' + 4xy' + 2y = \sec^2 x$, given that $y = 0$ and $y' = 1$ when $x = 0$

4. Define stable, unstable, stable &attractive points with suitable structure.

Section C

Attempt any FOUR: (5x4=20)

1. Apply the method of variation of parameters to solve differential the equation $x^2 y_2 + xy_1 - y = x^2 e^x$

2. Construct the Green's function for the homogenous BVP $\frac{d^4 y}{dx^4} = 0$, $y(0) = y'(0) = y'(1) = 0$

3. Find the real general solution of the following system & show the details $y'_1 = -8y_1 - 2y_2, y'_2 = 2y_1 - 4y_2$.

4. Transform the problem $\frac{d^2y}{dx^2} + y = x, y(0) = 1, y'(1) = 0$ to a Fredholm's integral equation.

5. If $f(x, y) = y^{2/3}$, show that Lipschitz condition is not satisfied in any region containing the origin and that the solution of the differential equation $\frac{dy}{dx} = f(x, y)$ satisfying the initial condition y=0 when x=0 is not unique.

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