15-12-2015

DOON UNIVERSITY, DEHRADUN

Final Semester Examination, First Semester, 2015

School of Physical Sciences

Class: M.Sc. Mathematics **Course: Finite Fields**

Semester: I **Course Code: MAC-401**

Time Allowed: 3Hours *Maximum Marks: 100*

Note: Attempt all six questions in Section A. Each question carries 3 marks.
Attempt any four questions in Section B. Each question carries 10 marks.
Attempt any three questions in Section C. Each question carries 14 marks.

SECTION: A

(Very Short Answer Type Questions)

(Marks: 6X3=18)

1. If H and K are two subgroups of a group G, then $H \cup K$ is a subgroup of G if and only if either $H \subset K$ or $K \subset H$.
2. Prove that every field is an integral domain.
3. Find all maximal ideals of Z_{12}.
4. Prove that every field is a P.I.D. Is the converse true?
5. Show that if R is an integral domain then $R[x]$ is an integral domain.
6. Define the followings:
   a. Primitive polynomial
   b. Reducible and Irreducible polynomials
   c. Characteristic of an integral domain
   d. Conjugate element in a group

SECTION: B

(Short Answer Type Questions)

(Marks: 4X10=40)

1. If G is a finite group, then $o[c(a)] = \frac{o(G)}{o(N(a))}$.
2. Prove that H is a normal subgroup of a group G iff the product of any two right cosets of H in G is a right coset of H in G.
3. Find all abelian groups, up to isomorphism, of order 720.
4. The necessary and sufficient conditions for a non-empty subset S of a ring R to be a subring of R are
   i. $a \in S,\ b \in S \Rightarrow a - b \in S$
   ii. $a \in S,\ b \in S \Rightarrow ab \in S$
5. If $f(x) = x^5 + 2x^3 + x^2 + 2x + 3$ and $g(x) = x^4 + x^3 + 4x^2 + 3x + 3$, then find the g.c.d. of $f(x)$ and $g(x)$ over $Z_5[x]$ and express it in the form of $m(x) . f(x) + n(x) . g(x)$.

SECTION: C

(Long Answer Type Questions)

(Marks: 3X14=42)

1. If H and K are two subgroups of a group G, then $O(HK) = \frac{O(H)O(K)}{O(H \cap K)}$.
2. If R is a commutative ring with unity, then prove that an ideal M of R is maximal ideal iff R/M is a field.
3. If f is a homomorphism of G onto G' with kernel K, then prove that $\frac{G}{K} \approx G'$.
4. Prove that every Euclidean domain is a Principal ideal domain.