17-12-2015

Integ. M.Sc.-I SEMESTER EXAMINATION 2015-16

Department of Mathematics, SOPS, Doon University Dehradun

Core Course-II, Algebra (MAC-102)

Time: 03 Hour **Total Marks: 100**

Note: (i) Attempt ALL the questions. (ii) Do neat and clean work.

Section A

Attempt any FOUR: **(4x5=20)**

1. Show that U=W, where U and W are the following subspaces of R^3: $U = span\{(1,1,-1),(2,3,-1),(3,1,-5)\}$
   $W = span\{(1,-1,-3)(3,-2,-8)(2,1,-3)\}$
2. Reduce the matrix $A = \begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$ to the diagonal form.
3. Find a real 2x2 symmetric matrix A with eigenvalues $\lambda_1 = 2$ *and* $\lambda_2 = 3$ and eigenvector X= (1,2) belonging to $\lambda_1 = 2$. Also find a matrix B for which $B^2 = A$
4. Let $G: R^3 \to R^3$ be the linear mapping defined by $G(x,y,z) = (x+2y-z, y+z, x+y-2z)$ find a basis and the dimension of (a) Im(G) (b) Ker(G).
5. Suppose A and B are orthogonal matrices. Show that A', A^{-1}, AB are also orthogonal.

Section B

Attempt any FOUR: **(4x5=20)**

1. Show that if $\lambda_1, \lambda_2 \dots\dots\dots \lambda_n$ *are the latent roots of the matrix* A, *than* A^3 has the latent roots ${\lambda_1}^3, {\lambda_2}^3, \dots\dots {\lambda_n}^3$.
2. If $a \equiv b (mod n)$ prove that $\gcd(a,n) = \gcd(b,n)$.
3. Show that the union of two subspaces may not be a subspace.
4. Show that the system of linear equation $-2x+y+z = a; x-2y+z = b; x+y-2z = c$ has no solution unless $a+b+c = 0$. In which case they have infinitely many solutions? Find these solutions when $a = 1, b = 1, c = -2$
5. Find the eigenvalues and eigenvectors of $A = \begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$

Section C

Attempt any FOUR: **(4x5=20)**

1. Show that all the matrices similar to an invertible matrix are invertible. More generally, show that similar matrices have the same rank.
2. Let dim V=n, $T: V \to V$ be a LT, such that the range T= KerT, Show that n is even, prove that $T: R^2 \to R^2$, such that $T(x_1, x_2) = (x_2, 0)$ is such a LT.
3. Find the LT form $T: R^3 \to R^3$ which has its range the subspace spanned by $(1,0,-1)(1,2,2)$
4. Show that for any integer a, $a^3 \equiv 0,1$ *or* $8 (mod 9)$.

5. Find two matrices P and Q such that PAQ is in its normal form by the elementary transformation on the matrix A= $\begin{bmatrix} 2 & 3 & 4 & -1 \\ 5 & 2 & 0 & -1 \\ -4 & 5 & 12 & -1 \end{bmatrix}$.

Section D

Attempt any TWO: (2x10=20)

1. If in a Ring R with unity $(xy)^2 = x^2y^2$ $for\ all\ x, y \in R$ then show that r is commutative.
2. By the Principal of mathematical induction prove that $cos\theta + cos2\theta + cos3\theta + \cdots \ldots + cosn\theta = sin\frac{(n+1)}{2}\theta . cos\frac{n\theta}{2} . cosec\frac{\theta}{2} - 1$
3. Define composite numbers. Prove that if $2^n - 1$ is prime then n is prime.

Section E

Attempt ALL **(from 1-6:1mark & 7-8:2mark)**

1. If$|A| = 0$, *then at least one eingen value is zero.* *(True or False)*
2. Find the dimension of the space P_n
3. Define the subspace of a vector space. Give an example.
4. The vectors (1,1,-1,1) (1,-1,2, -1) and (3, 1, 0,1) are LI or LD.
5. If λ is an eigenvalue of a symmetric matrix then λ is real *(True or False)*
6. Determine a nonzero linear transformation$T: V_2 \rightarrow V_2$which maps all the vectors on the line x=y onto the origin.
7. Define a Ring. Give an example.
8. Let$f(x) = 1 + x^2, -1 \leq x \leq 1, and\ g(x) = \sqrt{x}, x > 0$, *Describe gof. Is fog defined?*. justify your answer. Change the domain of g so that fog can be defined. Then describefog.

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