19-12-2015

DOON UNIVERSITY, DEHRADUN
End Semester Examination, First Semester, 2015
School of Physical Sciences
M.Sc.(Mathematics)
Course: MAC-101: Calculus

Time Allowed: 3 Hours **Maximum Marks: 100**

Note:
Attempt all questions from Sections A.
Attempt any four questions from Sections B.
Attempt any two questions from Sections C.

Section: A ($5 \times 4 = 20$ Marks)

(1) Find all the asymptotes of the curve

$$(y^2 - x^2)(y - 2x) - 7xy + 3y^2 + 2(x^2 + x + y) + 1 = 0.$$

(2) $\oint_C y^2dx + x^2dy$ if C is the boundary of the region bounded by the semicircle $y = \sqrt{4 - x^2}$ and x-axis.

(3) Find the area bounded by the $r = a(1 + \cos\theta), a > 0$.

(4) Find the volume of the tetrahedron bounded by the planes $x + 2y + z = 2, x = 2y, x = 0$ and $z = 0$.

(5) Compute the work done in moving a particle in a force field $\mathbf{F} = 2xy\,\mathbf{i} - 3x\,\mathbf{j} - 5z\,\mathbf{k}$ along the curve $x = t, y = t^2 + 1, z = 2t^2$ from $(0, 1, 0)$ to $(1, 2, 2)$.

Section: B ($10 \times 4 = 40$ Marks)

(1) For the cardioid $r = a(1 - \cos\theta)$, prove

(i) $\phi = \dfrac{\theta}{2}$.

(ii) $p = 2a\sin^3\dfrac{\theta}{2}$.

(iii) $p^2 = \dfrac{r^3}{2a}$.

(iv) polar subtangent$=2a\dfrac{\sin^3\frac{\theta}{2}}{\cos\frac{\theta}{2}}$.

(2) Find the centroid of an octant of a solid sphere.

(3) Evaluate the line integral $\oint_C \mathbf{F}.d\mathbf{r}$ where $\mathbf{F} = e^x\sin y\,\mathbf{i} + e^x\cos y\,\mathbf{j}$ and C is the rectangle with vertices $(0, 0), (1, 0), (1, \pi/2), (0, \pi/2)$.

(4) Find the volume of region above the $xy-$plane and under the graph of

$$z = 2 - |x| - |y|.$$

(5) Find the center of the mass of the lamina whose boundary consists of the parabola $y = x^2$, and the line $y = 9$ and whose density varies as $\rho = x^2 + y$.

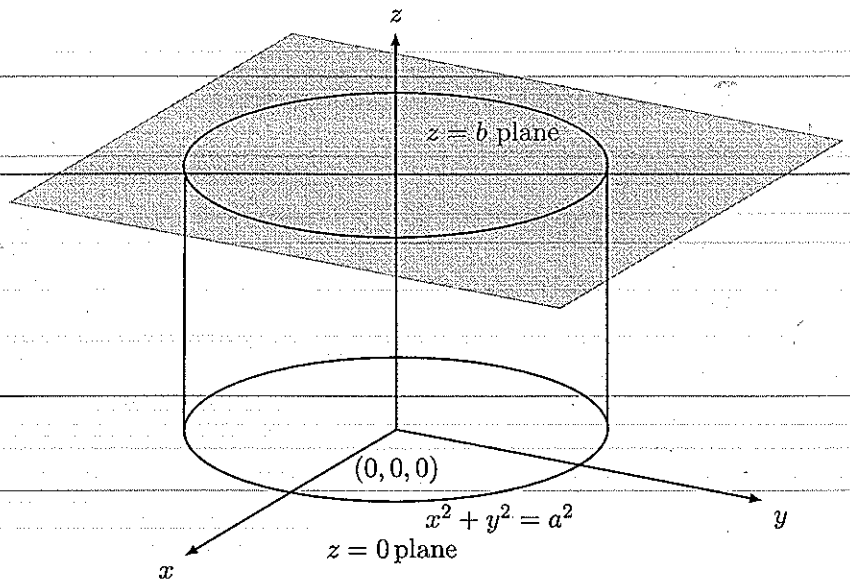

Fig. 1

Section: C ($20 \times 2 = 40$ Marks)

(1) (a) Prove, $\log(1 - x + x^2) = -x + \dfrac{x^2}{2} + \dfrac{2x^3}{3} + \dfrac{x^4}{4} - \cdots$

(b) Compute $\iiint x^5 y^3 z^4 \, dx \, dy \, dz$ where the integral extends over the region $x, y, z \geqslant 0$, $x^{2/3} + y^{2/3} + z^{2/3} = 8$.

(2) (a) Let $\mathbf{F}$ be the vector field $x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$ and D be the solid cylinder $x^2 + y^2 = a^2$ between $z = 0$ and $z = b$. Verify Gauss's theorem for this vector field and solid region(see figure 1).

(b) Find the area inside $r = 3\cos\theta$ and outside $r = 1 + \cos\theta$.

(3) (a) Evaluate $\oint_C (y^3 + \cos x)\,dx + (\sin y + z^2)\,dy + x\,dz$, where C is the oriented and parametrized closed curve $\mathbf{r}(t) = \cos t\,\mathbf{i} + \sin t\,\mathbf{j} + \sin 2t\,\mathbf{k}$.

(b) Find the moment of inertia of a rectangular plate with sides $2a$ and $2b$ about any edge.