Doon University, Dehradun
Final Examination, 2016
School of Social Science
MSc Semester – 3rd (Economics)
Course – SSEI -214
Mathematical Economics

Time Allowed: 3hrs   Maximum Marks: 50

(5x2 = 10 Marks)

Section - A

Attempt all questions

1. Show that $f(x) = \frac{1}{x-1}$ is continuous at x=1.

2. Evaluate: $\lim_{x \to 2} \frac{2x^2 - 7x + 6}{5x^2 - 11x + 2}$

3. If the net investment is a constant flow at I(t)=1000(dollar per year), what will be the total net investment during a year, from t=0 to t=1?

4. Solve: $\int (3x^3 + \sqrt{x} - \frac{1}{x}) dx$

5. Find the dual of the following primal:

   Min $Z_x = x_1 + x_2 + x_3$

   Subject to: (1) $x_1 - 3x_2 + 4x_3 = 5$

   (2) $x_1 - 2x_2 \leq 3$

   (3) $2x_2 - x_3 \geq 4$

   $x_1, x_2 \geq 0$, x_3 is unrestricted.

Section - B

Attempt any **five** questions

(5x3=15 Marks)

1. Determine consumer surplus and producer surplus under pure competition for the demand function $p=36-x^2$ and supply function $p=6+\frac{x^2}{4}$, where p is the price and x is quantity.

2. The total profit y, in rupees of a drug company from the manufacturing and sale of x drug bottles is given by,

   $$y = -(x^2/400) + 2x - 80$$

   (a) How many drug bottles must the company sell in order to achieve the maximum profit?

   (b) What is the profit per drug bottle when this maximum is achieved?

3. Discuss the maxima or minima for $f(x,y) = x^3 - 12xy + 8y^3$

4. If the marginal propensity to save (MPS) is the following function of income, $S'(Y) = 0.3 - 0.1Y^{-1/2}$, and if the aggregate savings S is nil when income is 81, find the saving function S(Y).

5. Write dual of the following primal problem:

$$\text{Maximize } Z_x = 3x_1 + x_2 + 3x_3 - x_4$$
$$\text{Subject to: } (1)\, 2x_1 - x_2 + 3x_3 + x_4 = 1$$
$$(2)\, x_1 + x_2 - x_3 + x_4 = 3 \text{ ; where } x_1, x_2, x_3, x_4 > 0$$

6. A function f(x) is defined in the interval (0,3) in the following way:

$f(x) = x^2$, when $0 < x < 1$

$f(x) = x$, when $1 \le x < 2$

$f(x) = \frac{1}{4}x^3$, when $2 \le x < 3$

Show the f(x) is continuous at x=2 & x=1

Section-C

Attempt any **five** questions   (5x5=25marks)

1. Obtain the solution of the following problem by using the method of Lagrangian multipliers:
   Minimize $z = x_1^2 + x_2^2 + x_3^2$
   Subject to: (1) $x_1 + x_2 + 3x_3 = 2$
   (2) $5x_1 + 2x_2 + x_3 = 5$

2. Consider the problem of designing an electronic device that consists of three main components. The components are arranged in series so that the failure of one of the will result in the failure of the whole device. Therefore, it is decided that the reliability (probability of failure) of the device should be increased by installing parallel units on each component. Each component may be installed in, at the most, three parallel units. The total capital (in thousand rupees) available for the device is 10. The following data is available:

Number of parallel units	Components					
	1		2		3	
	R_1	C_1	R_2	C_2	R_3	C_3
1	0.50	2	0.70	3	0.60	1
2	0.70	4	0.80	5	0.80	2
3	0.90	5	0.90	6	0.90	3

Where R_i, C_i (i=1,2,3) is the reliability and the cost of the ith component, respectively. Determine the number of parallel units that will maximise the total reliability of the system, without exceeding the given capital.

3. A company has decided to introduce a product in three phases. Phase 1 will feature making a special offer at a greatly reduced rate to attract the first- time buyers. Phase 2 will involve intensive advertising to persuade the buyers to continue purchasing at a regular price. Phase 3 will involve a follow up advertising and promotional campaign.

A total of ₹5 million has been budgeted for this marketing campaign. If m is the market share captured in Phase1, fraction f_2 of m is retained in Phase 2, and fraction f_3 of market share in Phase 2 is retained in Phase 3. The expected values of m, f_2 and f_3 at different levels of money expended are given below. How should the money be allocated to the three phases in order to maximise the final share?

Money spent	Effects on Market Share		
	m per cent	f_2	f_3
0	0	0.30	0.50
1	10	0.50	0.70
2	15	0.70	0.85
3	22	0.80	0.90
4	27	0.85	0.93
5	30	0.90	0.95

4. The total cost function of a firm is given by:
$$C= 0.04q^3-0.9q^2+10q+10$$

Find (a) Average cost (AC)

(b) Marginal cost (MC)

(c) Slope of AC

(d) Slope of MC

(e) Value of q at which average variable cost is minimum.

5. Obtain the solution of the following problem by using the method of Direct Substitution:
Minimize $Z= 3x_1^2 +x_2^2+x_3^2$
Subject to: $x_1+x_2+x_3=2$ and $x_1, x_2, x_3 \geq 0$

6. The cost of fuel for running a train is proportional to the cube of the speed generated in km per hour. When the speed is 12km per hour, the cost of fuel is Rs. 64 per hour. If other charges are fixed, namely Rs. 2000 per hour, find the most economical speed of train for running a distance of 100 km.