7-12-16

DOON UNIVERSITY, DEHRADUN

End Semester Examination, First Semester, 2016
Department of Physics, School of Physical Sciences
M.Sc. Physics 2 Year Programme
Course: PHC-402: Classical Mechanics

Time Allowed: 3 Hours **Maximum Marks: 50**

Instructions:

1) All questions are compulsory. Read the questions carefully and attempt each part.
2) Number the questions and their sub parts properly.
3) Write proper units wherever applicable.

SECTION: A

Marks: 4 x 2.5 = 10

1) What is the relation between the Hamiltonian in two system of coordinates that are related by canonical transformations?
2) Classify the different types of constraints based on dependence on velocity and time?
3) Use Poisson brackets to check whether the following transformation is canonical: $q = P + iaQ\ , p = \frac{P - iaQ}{2ia}$
4) At what speed does a meter stick move if its length is observed to shrink to 0.5m?

SECTION: B

Marks: 4 x 4 = 16

5) Consider a function f(q, p) of the coordinates q and momenta p. Use the Hamilton's equations to show that the time derivative of f can be written as :

$$\frac{df}{dt} = \frac{\partial f}{\partial q}\frac{\partial H}{\partial p} - \frac{\partial f}{\partial p}\frac{\partial H}{\partial q}$$

6) Consider a Cartesian coordinate x and the corresponding Lagrangian $L = \frac{m\dot{x}^2}{2} - V(x)$. Show that is x depends on another coordinate q and t, that is $x = x(q, t)$, then $L(q, \dot{q}, t)$ gives a Hamiltonian H that has the form,

$$H = T + V - m\left(\left(\frac{\partial x}{\partial q}\right)\left(\frac{\partial x}{\partial t}\right)\dot{q} + \left(\frac{\partial x}{\partial t}\right)^2\right)$$

Here T is the Kinetic energy of the body.

7) Check whether the following transformations are Canonical:
   a. $Q = \ln\left(\frac{1}{q}\ sinp\right), P = q\ cotp$
   b. $Q = \sqrt{2q}\ e^t\ cosp\ , P = \sqrt{2q}\ e^t\ sinp$ (2+2)
8) What are cyclic or ignorable coordinates? Show what special condition is satisfied by the generalized momenta corresponding to the cyclic coordinates using suitable equations? What is their special significance in canonical transformations? (1+2+1)

SECTION: C

Marks: 4 x 6 = 24

9) State and prove the Liouville's theorem. (1+5)
10) Derive the Hamilton-Jacobi equation. Write the equation that connects the Hamilton's principle function and the Hamilton's characteristic function, indicating clearly the arguments for each of the two functions. (4+2)
11) For the case of 1D Harmonic oscillator, show how to use the action-angle variables to obtain the frequency of oscillation for the system.
12) Using Hamilton's equation of motion, show that the Hamiltonian $H = \frac{p^2}{2m}\ e^{-rt} + \frac{1}{2}\ mw^2x^2\ e^{rt}$ leads to the equation of motion of a damped harmonic oscillator $\ddot{x} + r\dot{x} + w^2x = 0$.