9/12/16

DOON UNIVERSITY, DEHRADUN

End Semester Examination, First Semester, 2016-17

School of Physical Sciences

Class: Integrated M.Sc.(PHY, CHE, CS) **Semester: III**

Course: Applications of Algebra **Course Code: MAG-201**

Time Allowed: 3Hours *Maximum Marks: 100*

Note: Attempt all ten questions in Section A. Each question carries 2marks.
Attempt any eight questions in Section B. Each question carries 5 marks.
Attempt any four questions in Section C. Each question carries 10 marks.

SECTION: A
(Very Short Answer Type Questions)

(Marks: 10X2=20)

1. Express the matrix $\begin{bmatrix} 4 & 2 & -3 \\ 1 & 3 & -6 \\ -5 & 0 & -7 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix.
2. Find the rank of the matrix $\begin{bmatrix} 1 & 4 \\ 5 & 7 \end{bmatrix}$ using normal form method.
3. Identify the type of definiteness of the matrix $\begin{bmatrix} 3 & 2 & 3 \\ 4 & 5 & 3 \\ 2 & 4 & 5 \end{bmatrix}$.
4. Show that the two matrices A and $C^{-1}AC$ have the same characteristic roots.
5. Find whether or not the vectors $X_1 = (3, 1), X_2 = (2, 2), X_3 = (0, -4)$ are linearly dependent or independent.
6. Define binary operation on a set. Also, show that multiplication is a binary operation on the set $\{1, -1\}$ but not on the set $\{1, 2\}$.
7. Prove that the inverse of each element of a group is unique.
8. Find the order of the permutation $f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix}$, an element of a symmetric group S_4.
9. Define even and odd permutation. Also, determine whether the permutation $(12)(123)(1234)$ is even or odd.
10. Which of the following sets are group under multiplication modulo 11?
    (i) $A = \{1,10\}$ (ii) $B = \{1,8\}$.

SECTION: B
(Short Answer Type Questions)

(Marks: 8X5=40)

11. Find the rank of the matrix $A = \begin{bmatrix} 3 & 2 & 3 \\ 4 & 5 & 3 \\ 2 & 4 & 5 \end{bmatrix}$ using Echelon form method.
12. Find the inverse of the following matrix using Gauss Jorden method.
$$\begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}$$
13. Find the value of λ and μ so the following systems of equations have (a) no solution (b) a unique solution and (c) an infinite number of solutions
$$3x - y + \lambda z = 1$$
$$2x + y + z = 2$$
$$x + 2y - \lambda z = \mu$$

14. Determine the characteristic roots and the corresponding characteristic vectors of the matrix

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}.$$

15. Prove that the characteristic roots of an idempotent matrix are either zero or unity.
16. Classify the following quadratic form as positive definite, positive semi-definite, negative definite, negative semi definite, or indefinite.

$$f(x) = 2x_1^2 + x_2^2 - 3x_3^2 - 8x_2x_3 - 4x_1x_2 + 12x_1x_2 .$$

17. Show that the set $G = \{a + b\sqrt{2}: a, b \in Q\}$ is a group w.r.t. addition.
18. Define the order of a group. If in a group G, $a^5 = e,\ aba^{-1} = b^2\ \forall a, b \in G$, then find the order of b.
19. Show that, if a, b are any two elements of a group G, then $(ab)^2 = a^2b^2$ if and only if G is abelian.

SECTION: C

(Long Answer Type Questions)

(Marks:4X10=40)

20. If $A = \begin{bmatrix} 2 & -1 & 3 \\ -1 & 2 & 1 \\ 3 & 1 & -4 \end{bmatrix}$, then find two non-singular matrices P and Q such that $PAQ = I$. Hence find A^{-1}.
21. State and prove Cayley Hamilton theorem and verify it for the matrix $\begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}$.
22. Diagonalise the following matrix and obtain its modal matrix. Hence find A^5.

$$A = \begin{bmatrix} 3 & 1 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$$

23. Solve the following system of equations using Doolittle's LU factorization method.

$$x + 2y + 3z = 5$$
$$2x + 8y + 22z = 6$$
$$3x + 22y + 82z = -10$$

24. Let $G = \{(a, b): a \neq 0, b \in R\}$ and $*$ be a binary composition defined by

$$(a_1, b_1) * (a_2, b_2) = (a_1a_2, b_1a_2 + b_2).$$

Show that $(G,\ *)$ is a non-abelian group.